Adaptive Control

Chapter 9: Recursive plant model identification in closed loop

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Recursive plant model identification in closed loop

Abstract Iterative combination of the identification in closed loop and robust control redesign leads to a two time scale adaptive control system very appealing in practice. The chapter is dedicated to the presentation of recursive algorithms for plant identification in closed loop operation and their application. Two classes of algorithms will be presented, analyzed and evaluated experimentally: closed loop output error algorithms and filtered open loop recursive identification algorithms. Specific techniques for model validation in the context of identification in closed loop will also be presented. The performance of the various algorithm will be illustrated by simulation and by their application to the identification in closed loop and controller re-design of a flexible transmission control system.

Outline

- Identification in closed loop. Why ?
- An example (flexible transmission) and explanations
- Objectives of identification in closed loop
- Basic Schemes
- The CLOE Algorithms (closed loop output error)
- Properties of the algorithms
- Properties of the estimated models
- Validation of models identified in closed loop
- Iterative identification in closed loop and controller re-design
- Experimental results (flexible transmission)
- CLID – a toolbox for closed loop identification
- Use of open loop identification alg.for identification in closed loop
- Conclusions

Plant Identification in Closed Loop

Why ?

There are systems where open loop operation is not suitable (instability, drift, ..)

A controller may already exist (ex . : PID)

Re-tuning of the controller

a) to improve achieved performances
b) controller maintenance

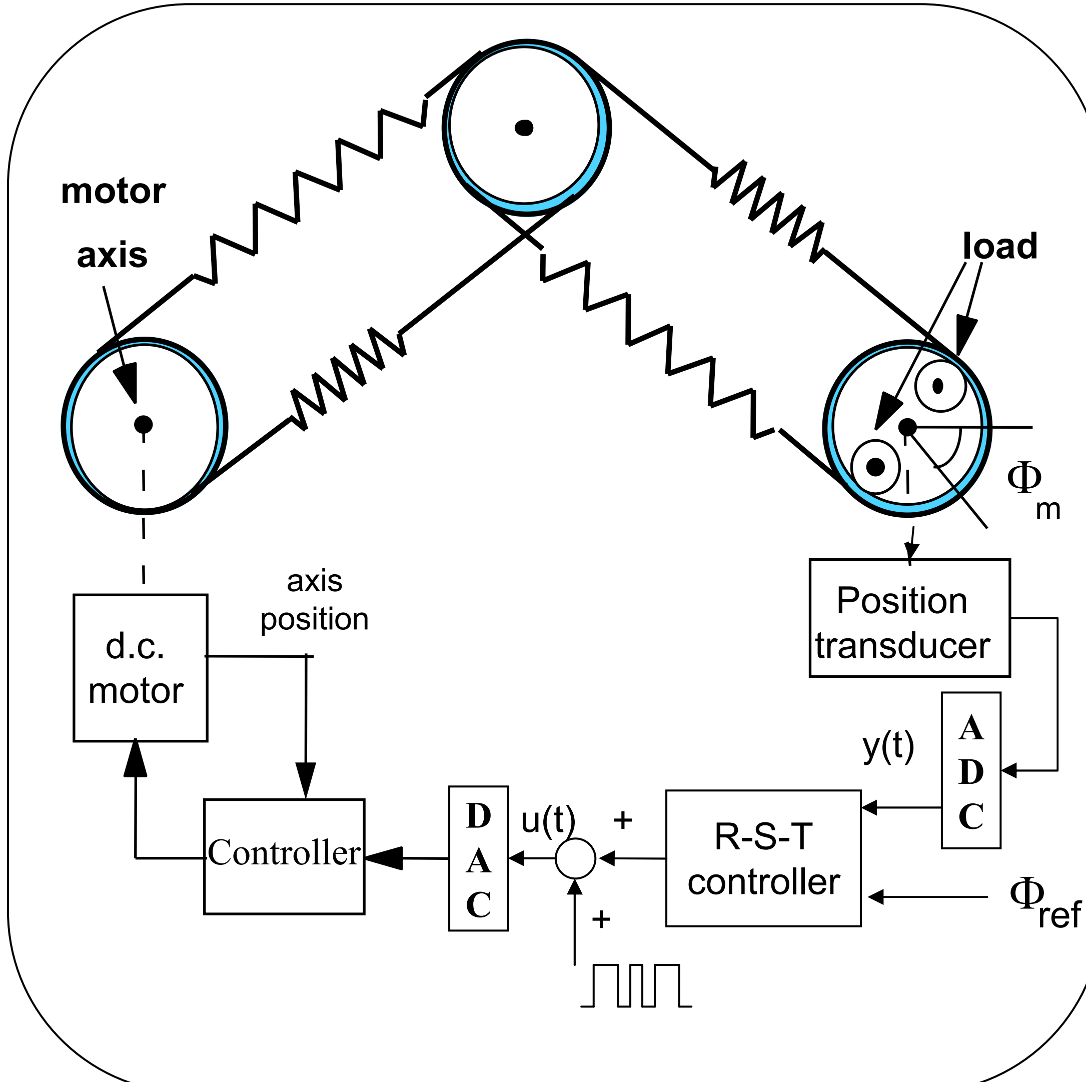
Iterative identification and controller redesign

May provide better « design » models

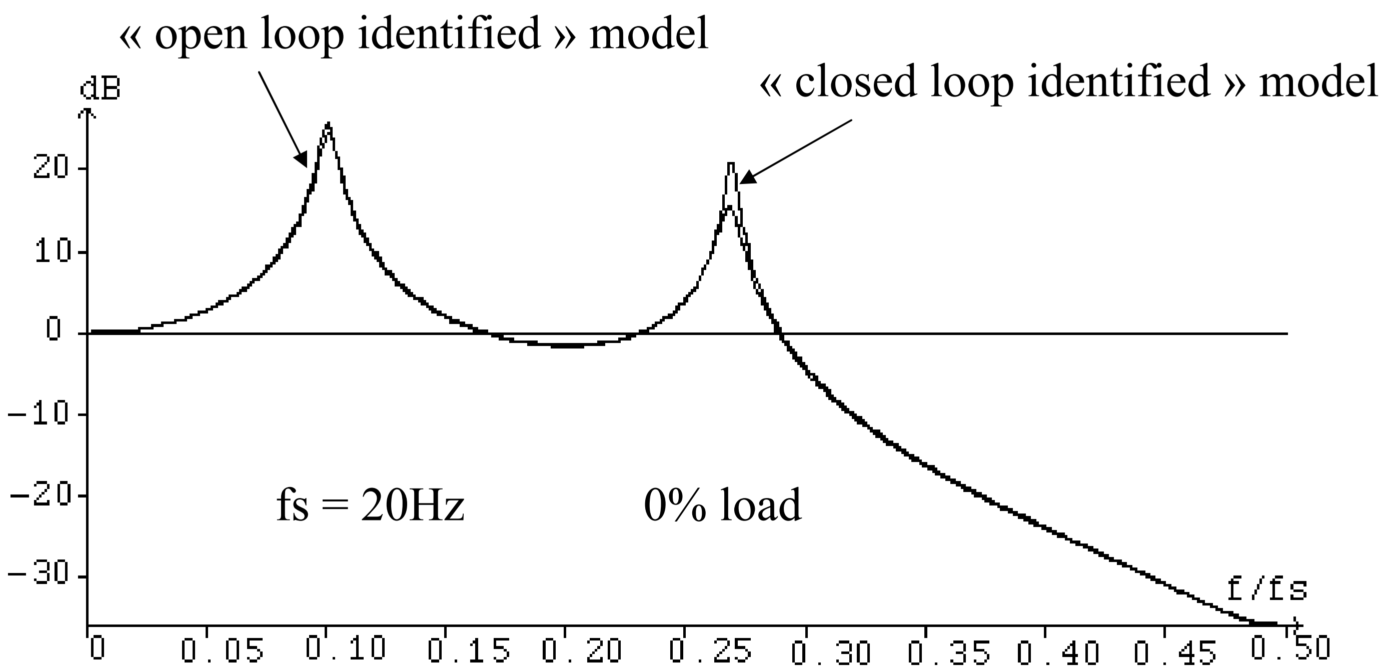
Cannot be dissociated from the controller and robustness issues

Identification in Closed Loop

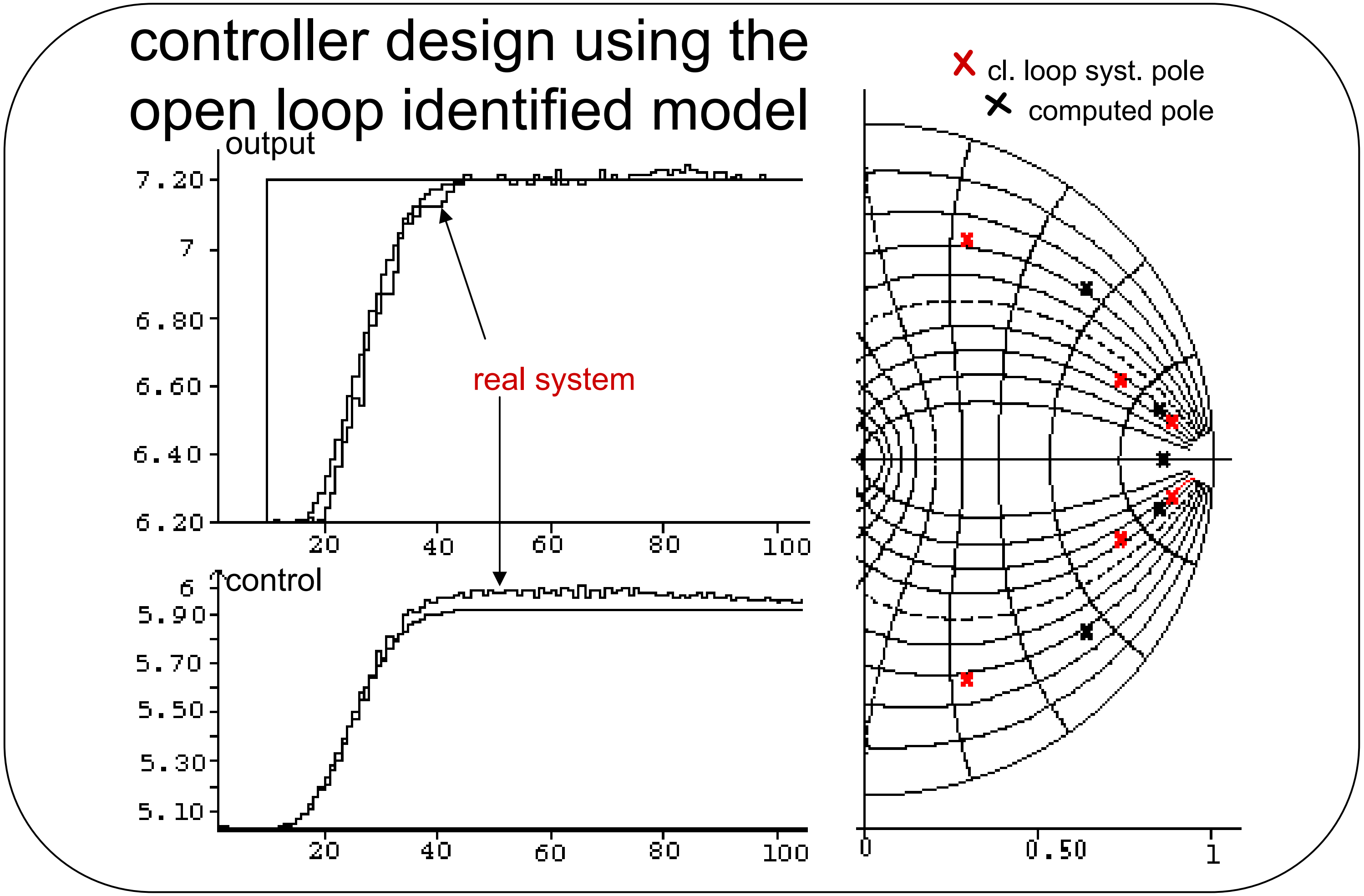
The flexible transmission



What is the *good* model (for control design) ?



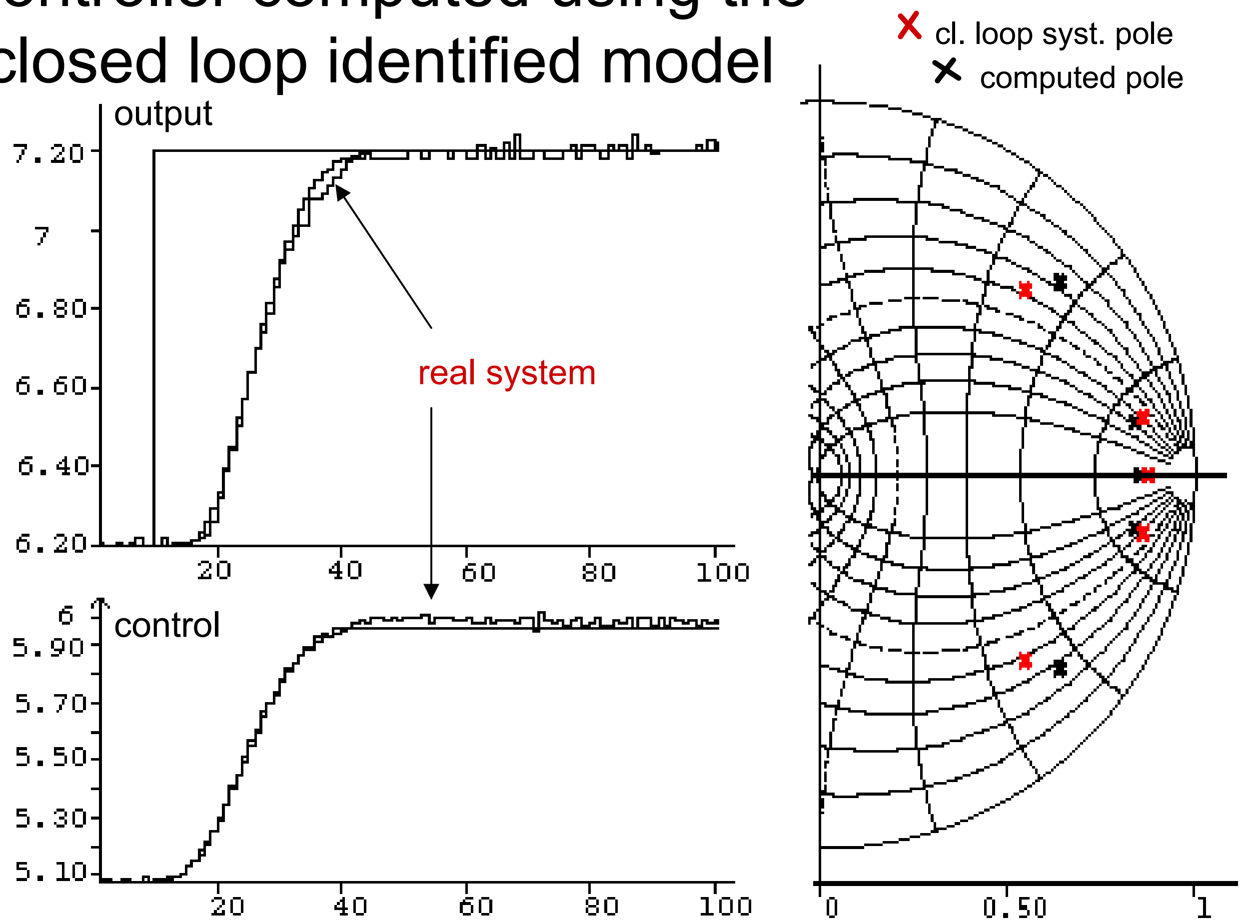
Benefits of identification in closed loop (1)



The pattern of *identified closed loop poles* is different from the pattern of *computed closed loop poles*

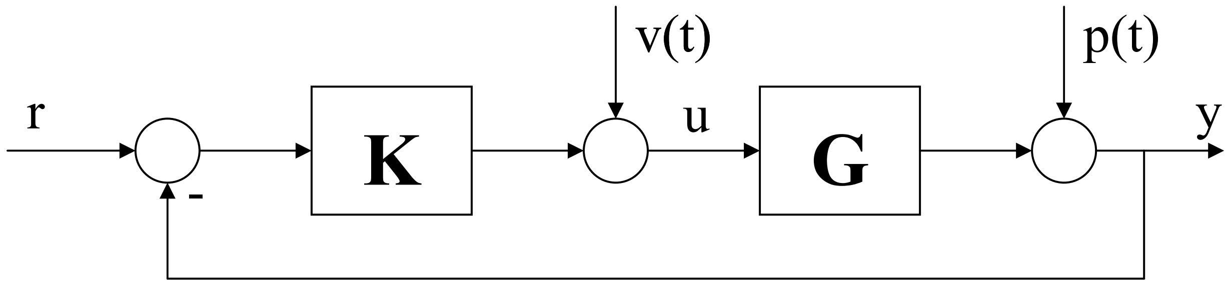
Benefits of identification in closed loop (2)

controller computed using the closed loop identified model



The *computed* and the *identified* closed loop poles are very close

Notations



$$G(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} \qquad K(q^{-1}) = \frac{R(q^{-1})}{S(q^{-1})}$$

Sensitivity functions :

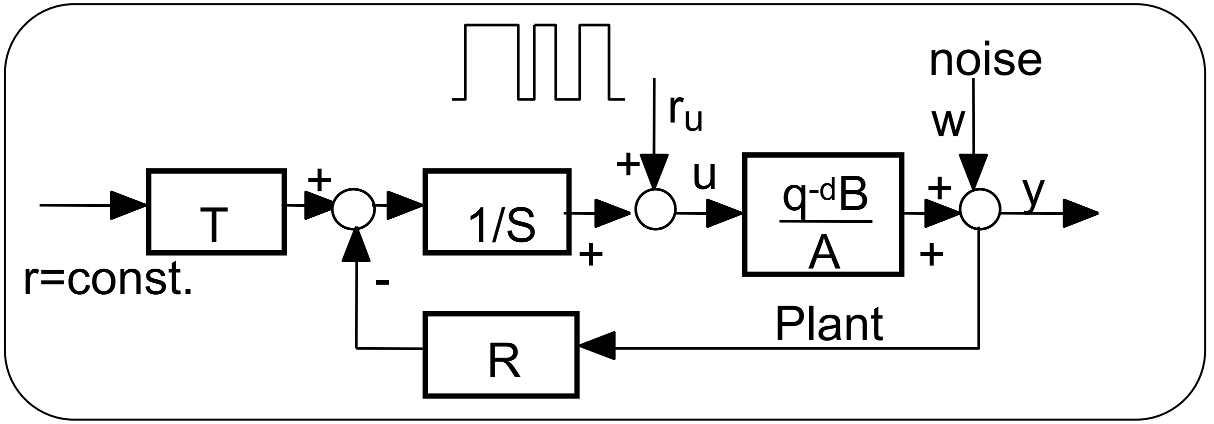
$$S_{yp}(z^{-1}) = \frac{1}{1+KG} \;;\; S_{up}(z^{-1}) = -\frac{K}{1+KG} \;;\; S_{yv}(z^{-1}) = \frac{G}{1+KG} \;;\; S_{yr}(z^{-1}) = \frac{KG}{1+KG}$$

Closed loop poles : $P(z^{-1}) = A(z^{-1})S(z^{-1}) + z^{-d}B(z^{-1})R(z^{-1})$

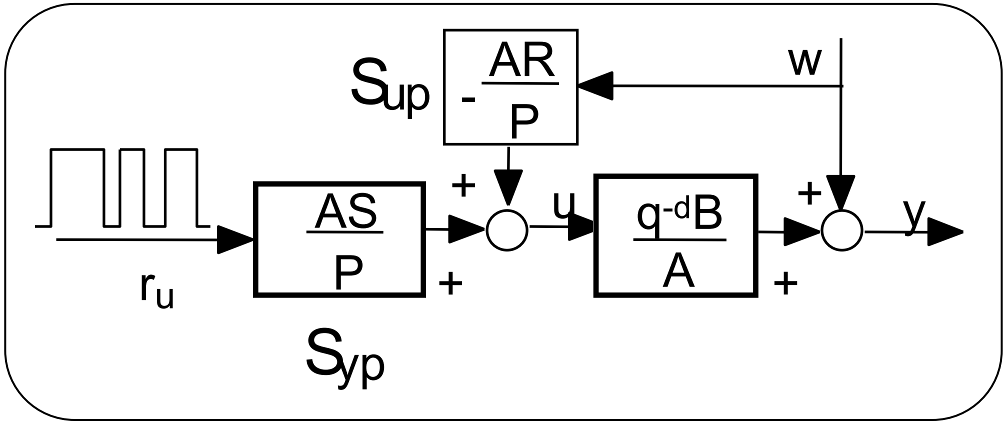
True closed loop system :(K,G), P, S_{xy}

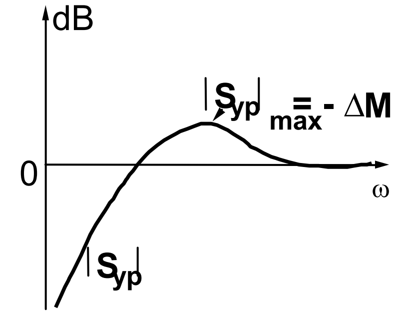
Nominal simulated(estimated) closed loop :$(K, \hat{G}), \hat{P}, \hat{S}_{xy}$

Identification in Closed Loop



Open loop interpetation





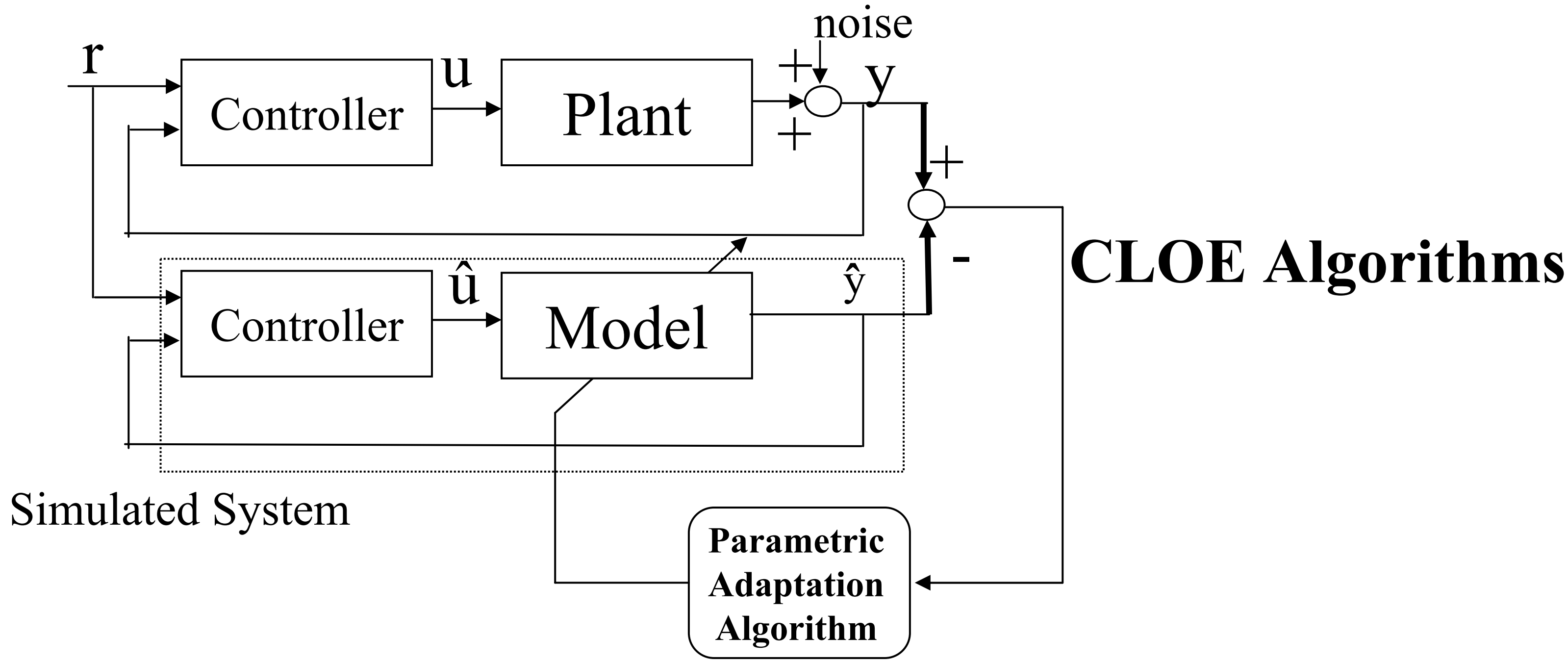
Objective : development of algorithms which:

- take advantage of the « improved » input spectrum
- are insensitive to noise in closed loop operation

Objective of the Identification in Closed Loop

(identification for control)

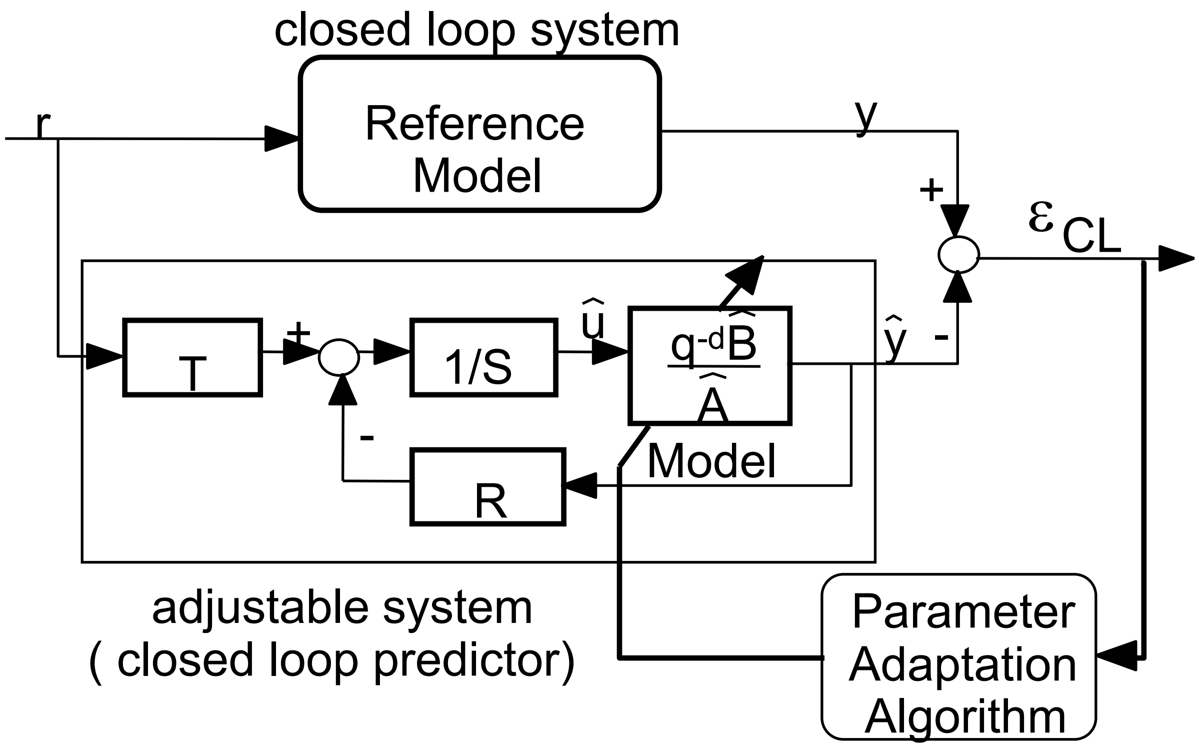
Find the « plant model » which minimizes the discrepancy between the « real » closed loop system and the « simulated » closed loop system.



Closed Loop Output Error

Identification in Closed Loop

- *M.R.A.S. point of view :*

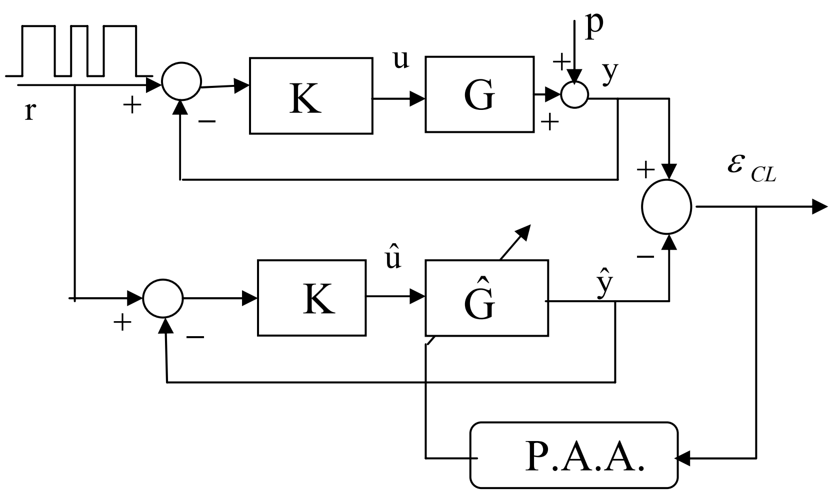


- *Identification point of view :*

A re-parametrized adjustable predictor of the closed loop

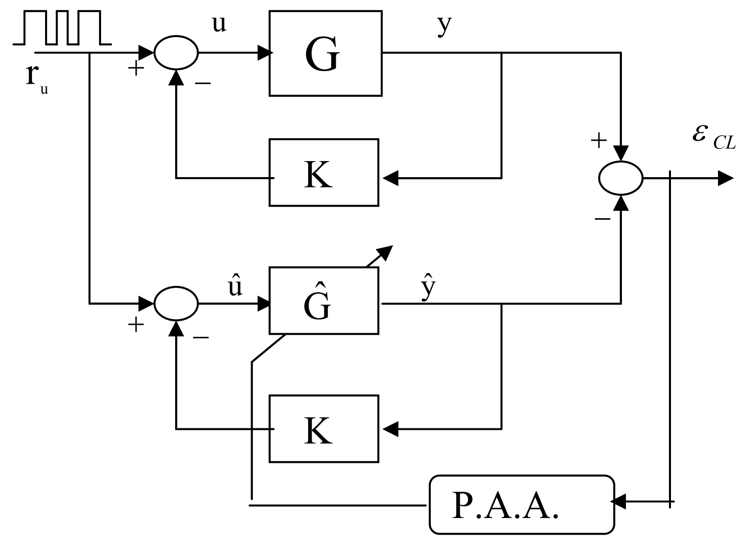
Closed Loop Output Error Identification Algorithms (CLOE)

Excitation added
to reference signal



$$u = -\frac{R}{S}y + \frac{R}{S}r \qquad \hat{u} = -\frac{R}{S}\hat{y} + \frac{R}{S}r$$

Excitation added
to controller output

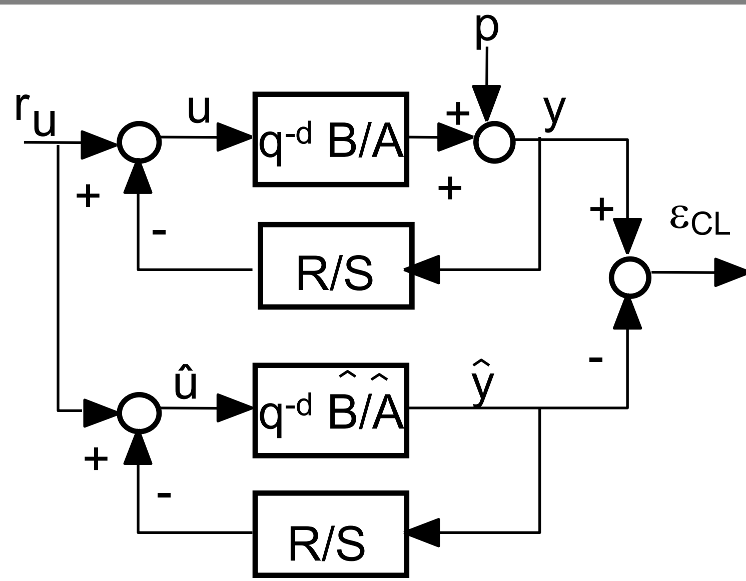


$$u = -\frac{R}{S}y + r_u \qquad \hat{u} = -\frac{R}{S}\hat{y} + r_u$$

Same algorithm but different properties of the estimated model!

Closed Loop Output Error Algorithms (CLOE)

Excitation added to the plant input



The closed loop system (for p = 0):

$$y(t+1) = -A^*(q^{-1})y(t) + B^*(q^{-1})u(t-d) = \theta^T\psi(t)$$

$$\theta^T = \left[a_1,\ldots a_{n_A}, b_1,\ldots b_{nB}\right]$$

$$\psi^T(t) = \left[-y(t),..-y(t-n_A+1), u(t-d),..u(t-d-n_B)\right]$$

$$u(t) = -\frac{R}{S}y(t) + r_u$$

CLOE

Adjustable predictor (closed loop)

Predicted output :

$$\hat{y}^0(t+1) = -\hat{A}^*(t,q^{-1})\hat{y}(t) + \hat{B}^*(t,q^{-1})\hat{u}(t-d) = \hat{\theta}^T(t)\phi(t) \quad \textit{a priori}$$

$$\hat{y}(t+1) = \hat{\theta}^T(t+1)\phi(t) \qquad \textit{a posteriori}$$

$$\hat{u}(t) = -\frac{R}{S}\hat{y}(t) + r_u$$

$$\hat{\theta}^T(t) = \left\lfloor \hat{a}_1(t),\ldots\hat{a}_{n_A}(t), \hat{b}_1(t),\ldots\hat{b}_{n_B}(t) \right\rfloor$$

$$\phi^T(t) = \left[-\hat{y}(t),..-\hat{y}(t-n_A+1), \hat{u}(t-d),..\hat{u}(t-d-n_B)\right]$$

Closed loop prediction (output) error

$$\varepsilon_{CL}^0(t+1) = y(t+1) - \hat{\theta}^T(t)\phi(t) = y(t+1) - \hat{y}^0(t+1) \qquad \textit{a priori}$$

$$\varepsilon_{CL}(t+1) = y(t+1) - \hat{\theta}^T(t+1)\phi(t) = y(t+1) - \hat{y}\ (t+1) \qquad \textit{a posteriori}$$

CLOE

The Parameter Adaptation Algorithm

$$\varepsilon_{CL}^{0}(t+1) = y(t+1) - \hat{\theta}^{T}(t)\phi(t) = y(t+1) - \hat{y}^{0}(t+1)$$

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t+1)\Phi(t)\varepsilon_{CL}^{0}(t+1)$$

$$F^{-1}(t+1) = \lambda_1(t)F^{-1}(t) + \lambda_2(t)\Phi(t)\Phi^{T}(t)\ ;\ 0 < \lambda_1(t) \leq 1\ ;\ 0 \leq \lambda_2(t) < 2$$

$$\Phi(t) = \phi(t)$$

Updating F(t):

$$F(t+1) = \frac{1}{\lambda_1(t)}\left[F(t) - \frac{F(t)\Phi(t)\Phi(t)^{T}F(t)}{\frac{\lambda_1(t)}{\lambda_2(t)} + \Phi(t)^{T}F(t)\Phi(t)}\right]$$

CLOE

The Parameter Adaptation Algorithm (alternative form)

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\Phi(t)\varepsilon_{CL}(t+1)$$

$$F^{-1}(t+1) = \lambda_1(t)F^{-1}(t) + \lambda_2(t)\Phi(t)\Phi^T(t) \; ; 0 < \lambda_1(t) \leq 1 \; ; 0 \leq \lambda_2(t) < 2$$

$$\varepsilon_{CL}(t+1) = \frac{\varepsilon^0_{CL}(t+1)}{1+\Phi^T(t)F(t)\Phi\ (t)}$$

$$\varepsilon^0_{CL}(t+1) = y(t+1) - \hat{\theta}^T(t)\phi(t) = y(t+1) - \hat{y}^0(t+1)$$

Mostly used for analysis purposes

CLOE Algorithms

CLOE $\boxed{\Phi(t) = \phi(t)}$

F-CLOE $\boxed{\Phi(t) = \dfrac{S(q^{-1})}{\hat{P}(q^{-1})}\phi(t)}$ $\hat{P} = \hat{A}(q^{-1})S(q^{-1}) + q^{-d}\hat{B}(q^{-1})R(q^{-1})$

AF-CLOE $\boxed{\Phi(t) = \dfrac{S(q^{-1})}{\hat{P}(q^{-1},t)}\phi(t)}$ $\hat{P}(q^{-1},t) = \hat{A}(q^{-1},t)S + q^{-d}\hat{B}(q^{-1},t)R$

$$\phi^T(t) = \left[-\hat{y}(t),..-\hat{y}(t-n_A+1),\hat{u}(t-d),..\hat{u}(t-d-n_B)\right]$$

Remarks :

• F-CLOE needs an « estimated model » for filtering. This can be an «open loop model» or a model identified with CLOE or AF-CLOE

• For AF-CLOE « inital estimation » for filtering can be $\hat{A} = 1,\ \hat{B} = 0$

CLOE Properties

Case 1: The plant model is in the model set
(i.e. the estimated model has the *good order)*

- The controller is constant
- An external excitation is applied
- Measurement noise independent w.r.t. the external excitation

Asymptotic unbiased estimates in the presence of noise
subject to a (mild) sufficient passivity condition

•*CLOE:* $S/P-\lambda/2$

•*F-CLOE:* $\hat{P}/P-\lambda/2$

Strictly positive real transfer fct.

$$\max_t \lambda_2(t) \le \lambda < 2$$

•*AF-CLOE:* *none (local)*

A basic result – deterministic environment

Consider the PAA

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\Phi(t)\varepsilon_{CL}(t+1)$$

$$F^{-1}(t+1) = \lambda_1(t)F^{-1}(t) + \lambda_2(t)\Phi(t)\Phi^T(t)\,;\, 0 < \lambda_1(t) \leq 1\,;\, 0 \leq \lambda_2(t) < 2$$

Assume that the *a posteriori* prediction error satisfies:

$$\boxed{\varepsilon_{CL}(t+1) = H(q^{-1})\left[\theta - \hat{\theta}(t+1)\right]^T \Phi(t)}$$

If: $\boxed{H(z^{-1}) - \frac{\lambda}{2} = S.P.R.;\ \max_t \lambda_2(t) \leq \lambda < 2}$

Then: $\lim_{t\to\infty} \varepsilon_{CL}(t+1) = \lim_{t\to\infty} \varepsilon^0_{CL}(t+1) = 0$

$$\left\|\Phi(t)\right\| < \infty; \forall t$$

for any initial conditions

CLOE analysis –Deterministic environment

(*a posteriori* prediction error equations)

CLOE $$\varepsilon_{CL}(t+1) = \frac{S}{P}\left[\theta - \hat{\theta}(t+1)\right]^T \phi(t)$$

F-CLOE $$\varepsilon_{CL}(t+1) \approx \frac{\hat{P}}{P}\left[\theta - \hat{\theta}(t+1)\right]^T \frac{S}{\hat{P}}\phi(t)$$

AF-CLOE $$\varepsilon_{CL}(t+1) \approx \frac{\hat{P}(t)}{P}\left[\theta - \hat{\theta}(t+1)\right]^T \frac{S}{\hat{P}(t)}\phi(t)$$

$\Phi(t)$

S.P.R. conditions for prediction error convergence

To have in addition « parameter convergence » one needs a persistent excitation (i.e.: rich input like P.R.B.S.)

A basic result – stochastic environment

$$\lambda_1 = 1 (or \lim_{t\to\infty} \lambda_1(t) = 1)$$

Assume that for any $\hat{\theta}$ along the trajectories of the algorithm:

$$\varepsilon_{CL}(t+1,\hat{\theta}) = H(q^{-1})\left[\theta - \hat{\theta}\right]^T \Phi(t,\hat{\theta}) + w'(t+1)$$

and: $E\left\{\Phi(t,\hat{\theta})w'(t+1)\right\} = 0$ (regressor and noise are uncorrelated)

If: $\boxed{H(z^{-1}) - \frac{\lambda}{2} = S.P.R.;\ \max_t \lambda_2(t) \le \lambda < 2,\ \lambda_2(t) > 0}$

Decreasing adaptation gain

Then: $Prob\left\{\lim_{t\to\infty} \hat{\theta}(t) \in D_C\right\} = 1$

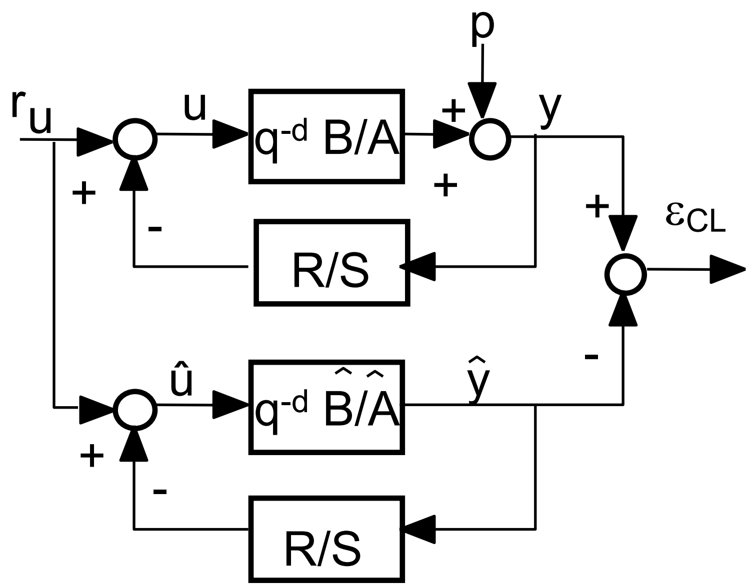
where: $D_C = \left\{\hat{\theta} : \left[\theta - \hat{\theta}\right]^T \Phi(t,\hat{\theta}) = 0\right\}$

Remark : With « persistent excitation » $D_C = \theta$

CLOE analysis – Stochastic environment

$w \neq 0$

Noisy case



The closed loop system:

$$y(t+1) = -A^*(q^{-1})y(t) + B^*(q^{-1})u(t-d) + Ap(t+1)$$

The a posteriori prediction error equation:

$$\varepsilon_{CL}(t+1) = \frac{S}{P}\left[\theta - \hat{\theta}(t+1)\right]^T \phi(t) + \frac{AS}{P} p(t+1)$$

CLOE analysis – Stochastic environment

The a posteriori prediction error equation:

$$\varepsilon_{CL}(t+1) = \frac{S}{P}\left[\theta - \hat{\theta}(t+1)\right]^T \phi(t) + \frac{AS}{P} p(t+1)$$

The « frozen » error equation:

$$\varepsilon_{CL}(t+1,\hat{\theta}) = \underbrace{\frac{S}{P}}_{H}\left[\theta - \hat{\theta}\right]^T \phi(t) + \underbrace{\frac{AS}{P} p(t+1)}_{w'}$$

$$E\left\{\phi(t,\hat{\theta})w'(t+1)\right\} = 0$$

(the input and output of the estimated model do not depend on the noise for fixed estimated parameters)

Convergence condition:

$S/P - \lambda/2 =$ S.P.R $\qquad \max_t \lambda_2(t) \le \lambda < 2$

CLOE Properties

Case 2: The plant model is not in the model set
(ex.: the estimated model has a *lower order)*

Basic idea for analysis of identification algorithms(Ljung) :
Convert time domain minimization criterion in frequency domain criterion using Parseval th. and Fourrier transforms

See the next slides

Analysis of identification algorithms in the frequency domain

(properties of the estimated model)

Identification criterion (time domain)

$$\hat{\theta}^* = \arg \min_{\hat{\theta} \in D} \lim_{t \to \infty} \frac{1}{N} \sum_{1}^{N} \varepsilon^2(t, \hat{\theta}) \approx \arg \min_{\hat{\theta} \in D} E\left\{\varepsilon^2(t, \hat{\theta})\right\}$$

domain of admissible parameters

Assumption: $\lim_{t \to \infty} \frac{1}{N} \sum_{1}^{N} \varepsilon^2(t, \hat{\theta}) < \infty$

Identification criterion (frequency domain)

$$\hat{\theta}^* = \arg \min_{\hat{\theta} \in D} \frac{1}{2\pi} \int_{-\pi}^{\pi} \phi_{\varepsilon}(e^{j\omega}, \hat{\theta}) d\omega$$

Spectral density of prediction error

Criterion minimized by CLOE algorithms

A recursive identification algorithm minimizes a criterion of the form:

$$\min_{\hat{\theta}\in D} \lim_{t\to\infty} \frac{1}{N} \sum_{1}^{N} \varepsilon^2(t,\hat{\theta}) \qquad (+)$$

if:

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\Phi(t)\varepsilon(t+1) = \hat{\theta}(t) + F(t)\left[-\frac{1}{2} grad\ \varepsilon^2(t+1)\right]$$

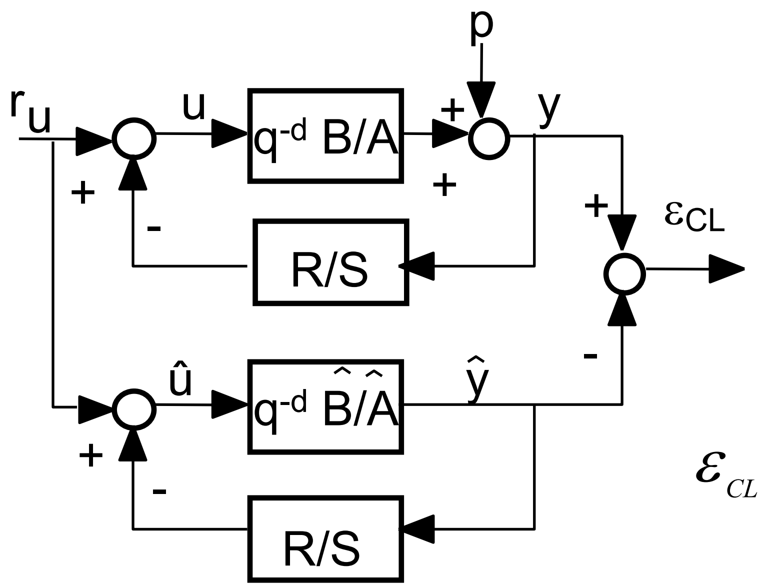
For CLOE algorithms: $\varepsilon_{CL}(t+1) = \frac{S}{P}\left[\theta - \hat{\theta}(t+1)\right]^T \phi(t)$

$$\frac{1}{2} grad\ \varepsilon^2(t+1) = \frac{\partial \varepsilon(t+1)}{\partial \hat{\theta}(t+1)} \varepsilon(t+1)$$

$$\frac{\partial \varepsilon(t+1)}{\partial \hat{\theta}(t+1)} = -\frac{S}{P}\phi(t) \approx -\Phi_{AF-CLOE}$$

AF-CLOE minimizes a criterion of the form (+)
(CLOE and F-CLOE achieve approximatively the same optimization)

CLOE – (excitation added to controller output)



$$r_u \to y = S_{yv}$$

$$r_u \to \hat{y} = \hat{S}_{yv}$$

$$S_{yv} = \frac{G}{1+KG}$$

$$\hat{S}_{yv} = \frac{\hat{G}}{1+K\hat{G}}$$

$$\varepsilon_{CL} = [S_{yv} - \hat{S}_{yv}]r_u + S_{yp}p$$

$\phi_{r_u}(\omega) =$ Spectral density of external excitation

$\phi_p(\omega) =$ Spectral density of the measurement noise

r_u and p are independent $\quad (\phi_{pr_u}(\omega) = 0)$

Properties of the Estimated Model (1)

Excitation added to controller output

$$\hat{\theta}^* = \arg\min_{\theta} \int_{-\pi}^{\pi} [\left|S_{yv} - \hat{S}_{yv}\right|^2 \phi_{r_u}(\omega) + \left|S_{yp}\right|^2 \phi_p(\omega)] d\omega$$

$$= \arg\min_{\theta} \int_{-\pi}^{\pi} \left|S_{yp}\right|^2 [\left|G - \hat{G}\right|^2 \left|\hat{S}_{yp}\right|^2 \phi_{r_u}(\omega) + \phi_p(\omega)] d\omega$$

- $\hat{G}$ will minimize the 2 norm between the true sensitivity function and the sensitivity function of the closed loop estimator when r(t) is a white noise (PRBS)

-Plant -model error heavily weighted by the sensitivity functions

-The noise does not affect the asymptotic estimation

Properties of the Estimated Model (2)

Excitation added to reference signal

$$\hat{\theta}^* = \arg\min_{\theta} \int_{-\pi}^{\pi} [\left|S_{yp} - \hat{S}_{yp}\right|^2 \phi_r(\omega) + \left|S_{yp}\right|^2 \phi_p(\omega)] d\omega$$

$$= \arg\min_{\theta} \int_{-\pi}^{\pi} \left|S_{yp}\right|^2 [\left|G - \hat{G}\right|^2 \left|\hat{S}_{up}\right|^2 \phi_r(\omega) + \phi_p(\omega)] d\omega$$

- $\hat{G}$ will minimize the 2 norm between the true sensitivity function and the sensitivity function of the closed loop estimator when r(t) is a white noise (PRBS)

-Plant -model error heavily weighted by the sensitivity functions

-The noise does not affect the asymptotic estimation

Properties of the Estimated Model (3)

Excitation added to reference signal

One has: $\left|S_{yp} - \hat{S}_{yp}\right| = \left|S_{yr} - \hat{S}_{yr}\right|$

Therefore one has also :

$$\hat{\theta}^* = \arg\min_{\theta} \int_{-\pi}^{\pi} [\left|S_{yr} - \hat{S}_{yr}\right|^2 \phi_r(\omega) + \left|S_{yp}\right|^2 \phi_p(\omega)] d\omega$$

$$= \arg\min_{\theta} \int_{-\pi}^{\pi} [\left|S_{yp} - \hat{S}_{yp}\right|^2 \phi_r(\omega) + \left|S_{yp}\right|^2 \phi_p(\omega)] d\omega$$

The differences with respect to the output sensitivity function and the complimentary sensitivity function are minimized

Identification in closed loop - Some remarks

- The quality of the identified model is enhanced in the critical frequency regions for control (compare with open loop id.)

$$\text{CLOE} \qquad \hat{\theta}^* = \arg\min_{\theta} \int_{-\pi}^{\pi} \left|S_{yp}\right|^2 \left[\left|G-\hat{G}\right|^2 \left|\hat{S}_{yp}\right|^2 \phi_r(\omega) + \phi_p(\omega)\right] d\omega$$

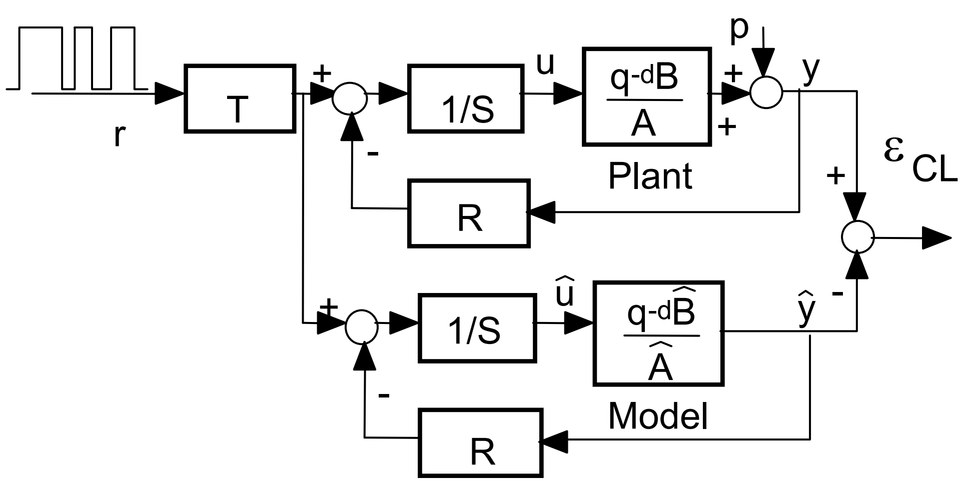
$$\text{OLOE} \qquad \hat{\theta}^* = \arg\min_{\theta} \int_{-\pi}^{\pi} \left[\left|G-\hat{G}\right|^2 \phi_r(\omega) + \phi_p(\omega)\right] d\omega$$

-Identification in closed loop can be used for **model reduction.**
The approximation will be good in the critical frequency regions for control.

Closed Loop Output Error Identification Algorithms (CLOE)

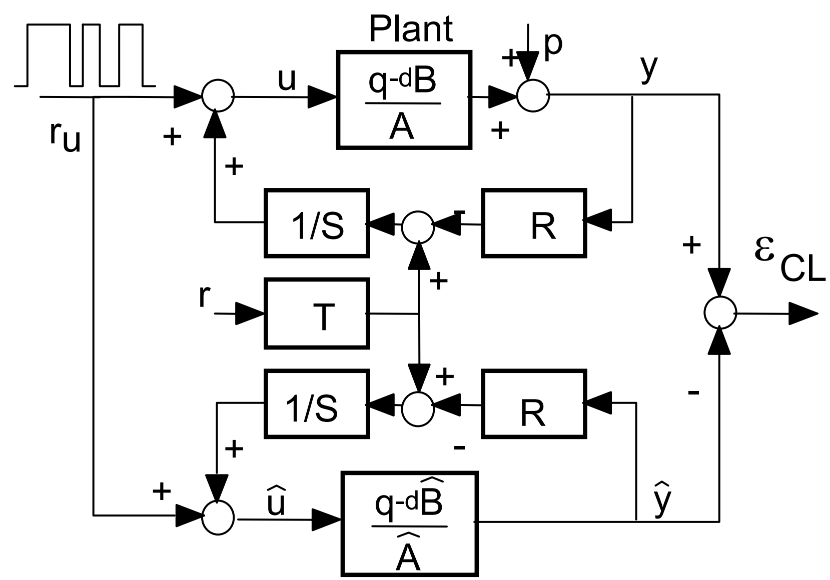
R-S-T Controller

Excitation added to reference signal



$$u = -\frac{R}{S}y + \frac{T}{S}r \qquad \hat{u} = -\frac{R}{S}\hat{y} + \frac{T}{S}r$$

Excitation added to controller output



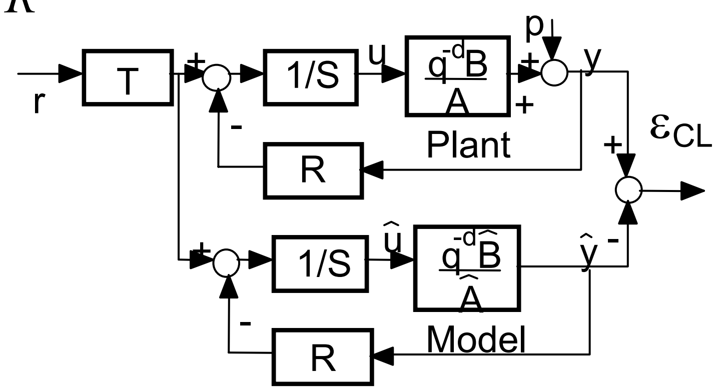
$$u = -\frac{R}{S}y + r_u \qquad \hat{u} = -\frac{R}{S}\hat{y} + r_u$$

Use of the prefilter T (R-S-T controller)

Difference between closed loop transfer functions (excitation through *T*)

$$\frac{BT}{AS+BR}-\frac{\hat{B}T}{\hat{A}S+\hat{B}R}=\frac{T}{R}\left[\frac{BR}{AS+BR}-\frac{\hat{B}R}{\hat{A}S+\hat{B}R}\right]=\frac{T}{R}\left[S_{yr}-\hat{S}_{yr}\right]$$

$$=\frac{T}{S}\left[\frac{BS}{AS+BR}-\frac{\hat{B}S}{\hat{A}S+\hat{B}R}\right]=\frac{T}{S}\left[S_{yv}-\hat{S}_{yv}\right]$$



Properties of the estimated model:

$$\hat{\theta}^{*}=\arg\min_{\theta}\int_{-\pi}^{\pi}\left[\left|S_{yr}-\hat{S}_{yr}\right|^{2}\left|\frac{T}{R}\right|^{2}\phi_{r}(\omega)+\left|S_{yp}\right|^{2}\phi_{p}(\omega)\right]d\omega$$

$$=\arg\min_{\theta}\int_{-\pi}^{\pi}\left[\left|S_{yv}-\hat{S}_{yv}\right|^{2}\left|\frac{T}{S}\right|^{2}\phi_{r}(\omega)+\left|S_{yp}\right|^{2}\phi_{p}(\omega)\right]d\omega$$

$T=S$ *Excitation added to the plant input*

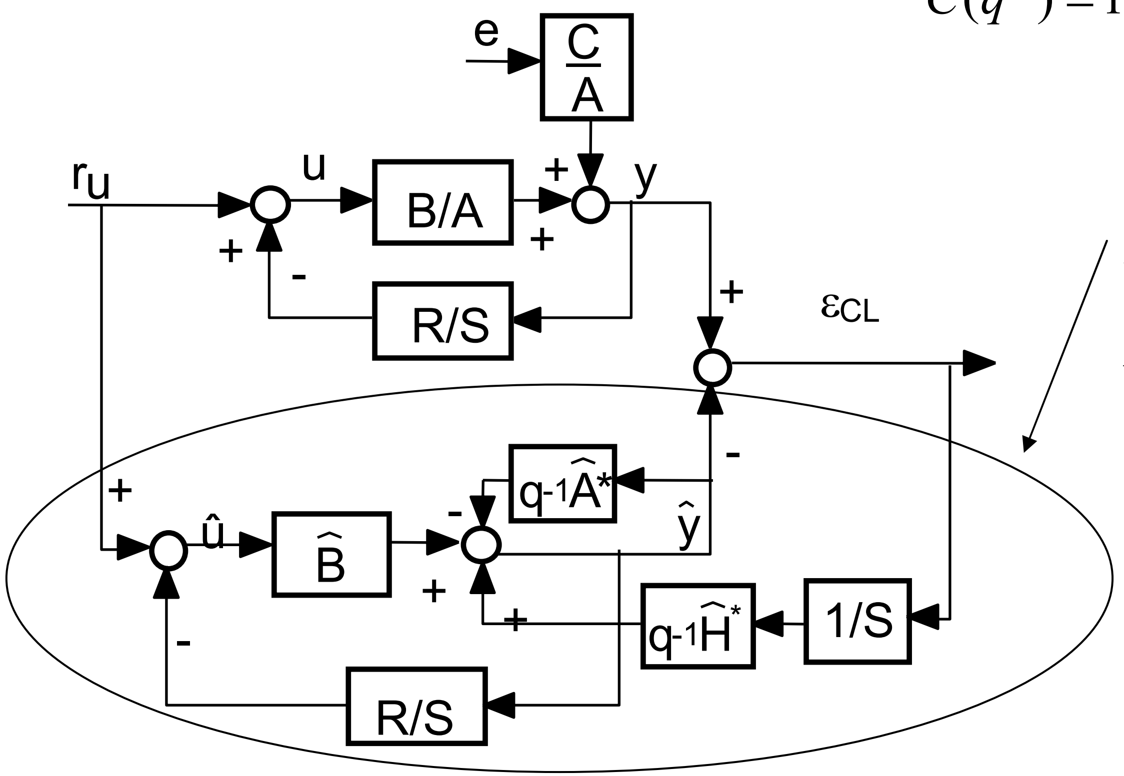
$T=R$ *Excitation added to the controller input (measure)*

Identification in Closed Loop of ARMAX Models

X-CLOE *Extended Closed Loop Output Error*

$$y(t+1) = -A^*(q^{-1})y(t) + B^*(q^{-1})u(t-d) + C^*e(t) + e(t+1)$$

$$C(q^{-1}) = 1 + q^{-1}C^*(q^{-1})$$



Optimal predictor for the closed loop when :

$\hat{A} = A; \hat{B} = B; \hat{C} = C$

$$H^* = C^*S - A^*S - B^*R;\ H = 1 + q^{-1}H^*;\ nH \cong nP$$

X-CLOE – the algorithm

Predicted output :

$$\hat{y}^\circ(t+1) = \hat{\theta}_e^T(t)\phi_e(t) \qquad \textit{a priori}$$

$$\hat{u}(t) = -\frac{R}{S}\hat{y}(t) + r_u$$

$$\hat{\theta}_e^T(t) = \left[\hat{a}_1(t),\ldots\hat{a}_{n_A}(t),\hat{b}_1(t),\ldots\hat{b}_{n_B}(t),\hat{h}_1(t),\ldots\hat{h}_{n_H}(t)\right]$$

$$\varepsilon_{CLf}(t) = \frac{1}{S}\varepsilon_{CL}(t)$$

$$\phi^T(t) = \left[-\hat{y}(t),.-\hat{y}(t-n_A+1),\hat{u}(t-d),.\hat{u}(t-d-n_B),\varepsilon_{CLf}(t),.\varepsilon_{CLf}(t-n_H+1)\right]$$

Closed loop prediction (output) error

$$\varepsilon_{CL}^0(t+1) = y(t+1) - \hat{\theta}_e^T(t)\phi_e(t) = y(t+1) - \hat{y}^0(t+1) \qquad \textit{a priori}$$

X-CLOE – the algorithm

The Parameter Adaptation Algorithm

$$\varepsilon^{0}_{CL}(t+1) = y(t+1) - \hat{\theta}_e^{T}(t)\phi_e(t) = y(t+1) - \hat{y}^{0}(t+1)$$

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t+1)\Phi(t)\varepsilon^{0}_{CL}(t+1)$$

$$F^{-1}(t+1) = \lambda_1(t)F^{-1}(t) + \lambda_2(t)\Phi(t)\Phi^{T}(t)\ ;\ 0 < \lambda_1(t) \leq 1\ ;\ 0 \leq \lambda_2(t) < 2$$

$$\Phi(t) = \phi_e(t)$$

X-CLOE Properties

Case 1: The plant model is in the model set

Deterministic case:
- Global convergence does not require any S.P.R. condition *(works allways)*

Stochastic case (noise)
- Asymptotic unbiased estimates
- Convergence condition: $1/C - \lambda/2 =$ S.P.R *(like in open loop for ELS and OEEPM)*

Case 2: The plant model is not in the model set

- Slightly less good « approximation » properties than CLOE
- Provides better results than « open loop » identification alg.

Validation of Models Identified in Closed Loop

Controller dependent validation !

1) Statistical Model Validation

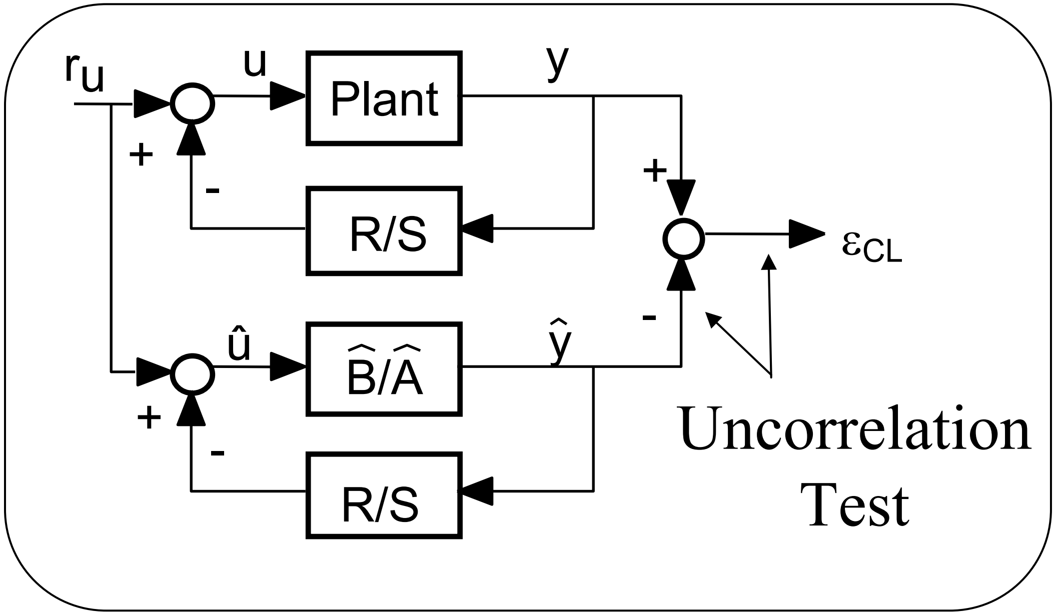
2) Pole Closeness Validation

3) Sensitivity Functions ClosenessValidation

4) Time Domain Validation

Identification in Closed Loop

Statistical Model Validation

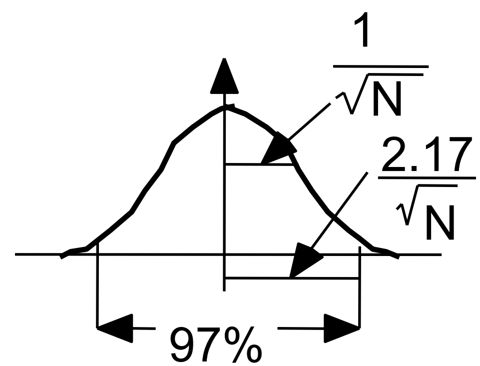


Controller dependent validation !

$$|RN(i)| \le \frac{2.17}{\sqrt{N}}\,;\; i \ge 1$$

normalized crosscorelations

number of data



$$N = 256 \rightarrow |RN(i)| \le 0.136$$

pratical value : $\boxed{|RN(i)| \le 0.15}$

« Uncorrelation » Test

$\{\varepsilon_{CL}(t)\}$: centered sequence of residual closed loop prediction errors

One computes:

$$R(i) = \frac{1}{N}\sum_{t=1}^{N}\varepsilon_{CL}(t)\hat{y}(t-i) \quad ; \quad i = 0,1,2,\ldots,i_{max} \qquad ; \quad i_{\max} = \max(n_A, n_B + d)$$

$$RN(i) = \frac{R(i)}{\left[\left(\frac{1}{N}\sum_{t=1}^{N}\hat{y}^2(t)\right)\left(\frac{1}{N}\sum_{t=1}^{N}\varepsilon_{CL}{}^2(t)\right)\right]^{1/2}} \quad ; \quad i = 0,1,2,\ldots,i_{max}$$

Remark: $RN(0) \neq 1$

Theoretical values: *RN(i) =0; i = 1, 2…*i_{max}

Real situation:
- Finite number of data
- Residual structural errors (orders, nonlinearities, noise)
- Objective: to obtain « good » simple models

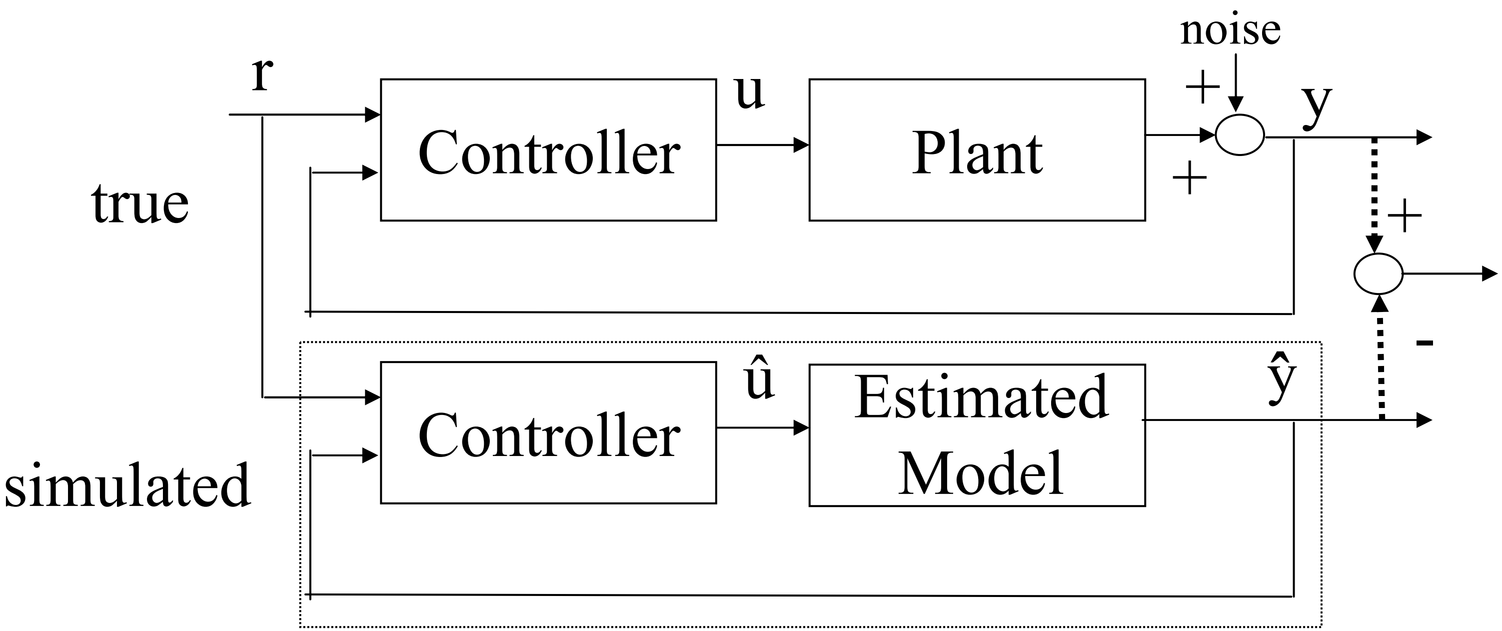
Validation criterion (N = number of data):

$$\boxed{|RN(i)| \leq \frac{2.17}{\sqrt{N}} \quad ; \quad i \geq 1}$$

or: $|RN(i)| \leq 0.15\,; \; i = 1,\ldots,i_{\max}$

Identification in Closed Loop

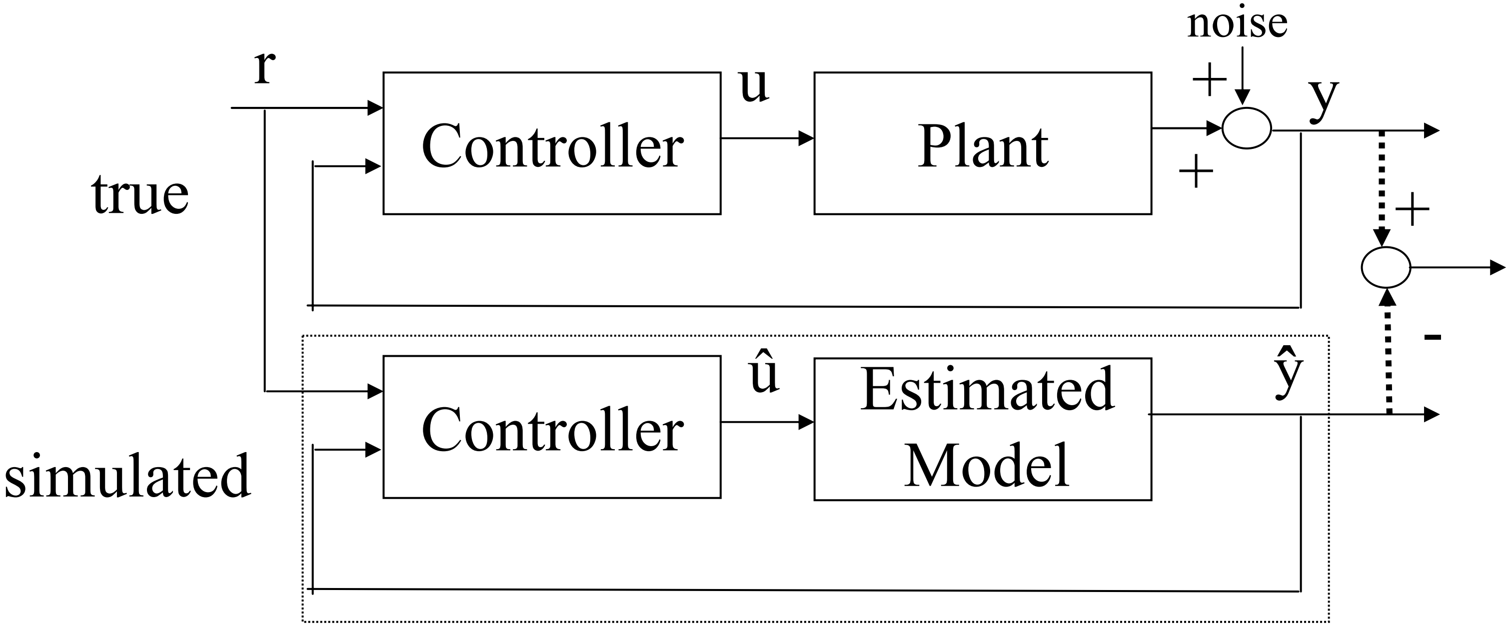
Pole Closeness Validation



If the estimated model is good, the poles of the « true » loop and of the « simulated » loop should be *close*

- *The poles of the « simulated » system can be computed*
- *The poles of the « true » system should be estimated*

Sensitivity Function Closeness Validation

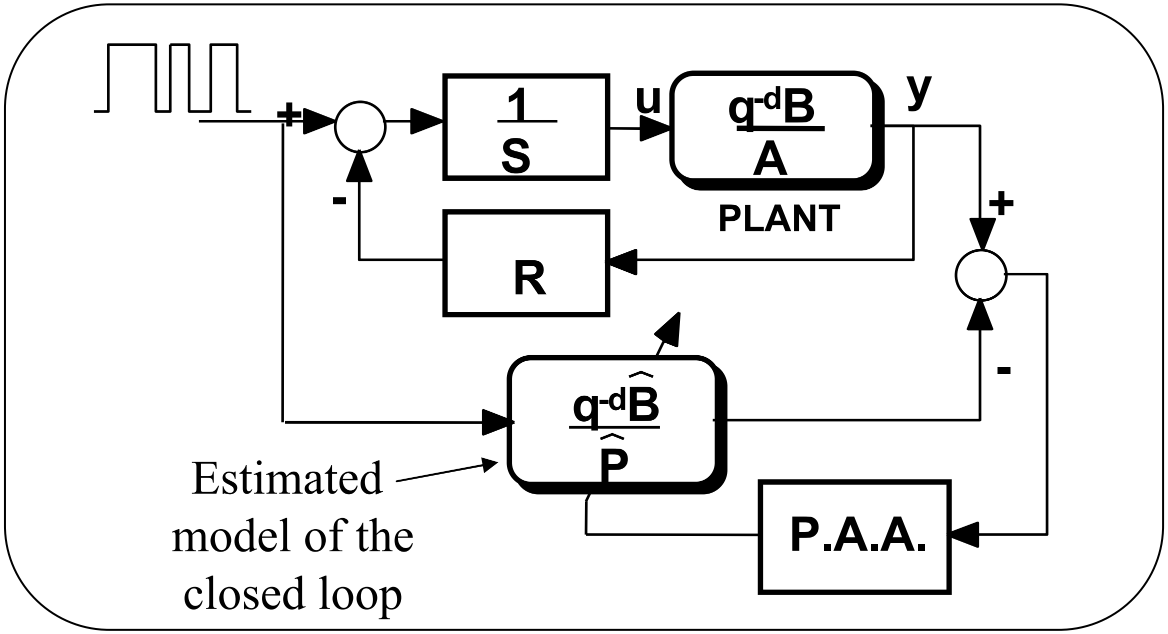


If the estimated model is good, the sensitivity functions of the « true » loop and of the « simulated » loop should be *close*

• *The sensitivity fct. of the « simulated » system can be computed*

• *The sensitivity fct. of the « true » system should be estimated*

Closed loop poles/Sensitivity functions Estimation



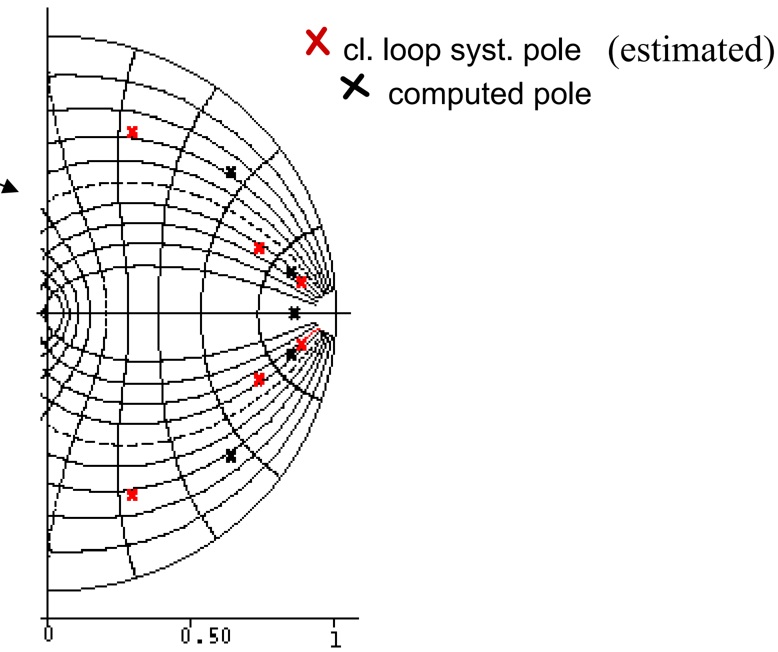
Rem:

- use of open loop identification algorithms
- same signals as those used for the identification of the *plant model* in closed loop operation
- attention to the selection of the order !

How to asses the poles/sensitivity fct. closeness ?

Poles:

- Patterns of the poles map
- Closeness of $1/\hat{P}$ and $1/\hat{\hat{P}}$



Sensitivity functions:

- Closeness of $\hat{S}_{xy}$ and $\hat{\hat{S}}_{xy}$

The closeness of two transfer functions can be measured by the « Vinnicombe distance » (ν gap) (min = 0, max = 1)
(*will be discussed later)*

$\hat{X}$: Closed loop transfer function computed with the estimated plant model

$\hat{\hat{X}}$: Estimated closed loop transfer function

Normalized distance between two transfer functions (G_1, G_2) (Vinnicombe distance or ν–gap)

The winding number:

$$wno(G) = n_{z_i}(G) - n_{p_i}(G)$$

Unstable zeros Unstable poles

wno(G)>0 wno(G)<0

$wno(G)$ = number of encirclements of the origin (winding number)
(+ : counter clock wise , - : clock wise)

One can compares transfer functions satisfying :

$$wno(1 + G_2^* G_1) + n_{p_i}(G_1) - n_{p_i}(G_2) - n_{P_1}(G_2) = 0 \qquad \{w\}$$

G^* = complex conjugate of G $\quad n_{P_1}(G_2)$ = number of poles on the unit circle

Normalized distance between two transfer functions (G_1, G_2) (Vinnicombe distance or ν–gap)

One assumes that {w} is satisfied.

Normalized difference :

$$\Psi\left[G_1(j\omega), G_2(j\omega)\right] = \frac{G_1(j\omega) - G_2(j\omega)}{\left(1 + \left|G_1(j\omega)\right|^2\right)^{1/2}\left(1 + \left|G_2(j\omega)\right|^2\right)^{1/2}}$$

Normalized distance (Vinnicombe distance or ν-gap) :

$$\boxed{\delta_\nu(G_1, G_2) = \left|\, \Psi\left[G_1(j\omega), G_2(j\omega)\right] \right|_{\max\limits_{\omega}} = \left\| \Psi\left[G_1(j\omega), G_2(j\omega)\right] \right\|_\infty \quad \text{for } \omega = 0 \text{ to } \pi f_s}$$

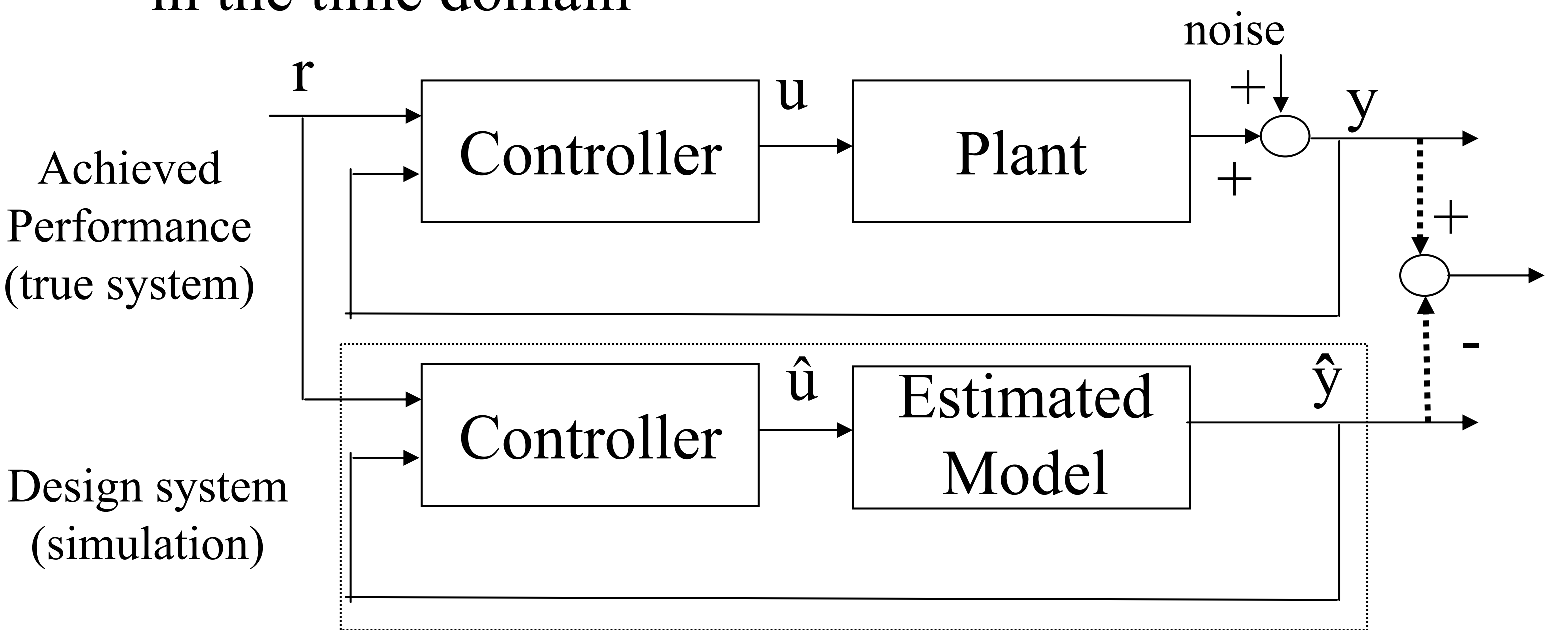
$$0 \leq \delta_\nu(G_1, G_2) < 1$$

If {w} is not satisfied : $\delta_\nu(G_1, G_2) = 1$

Identification in Closed Loop

Time Domain Validation

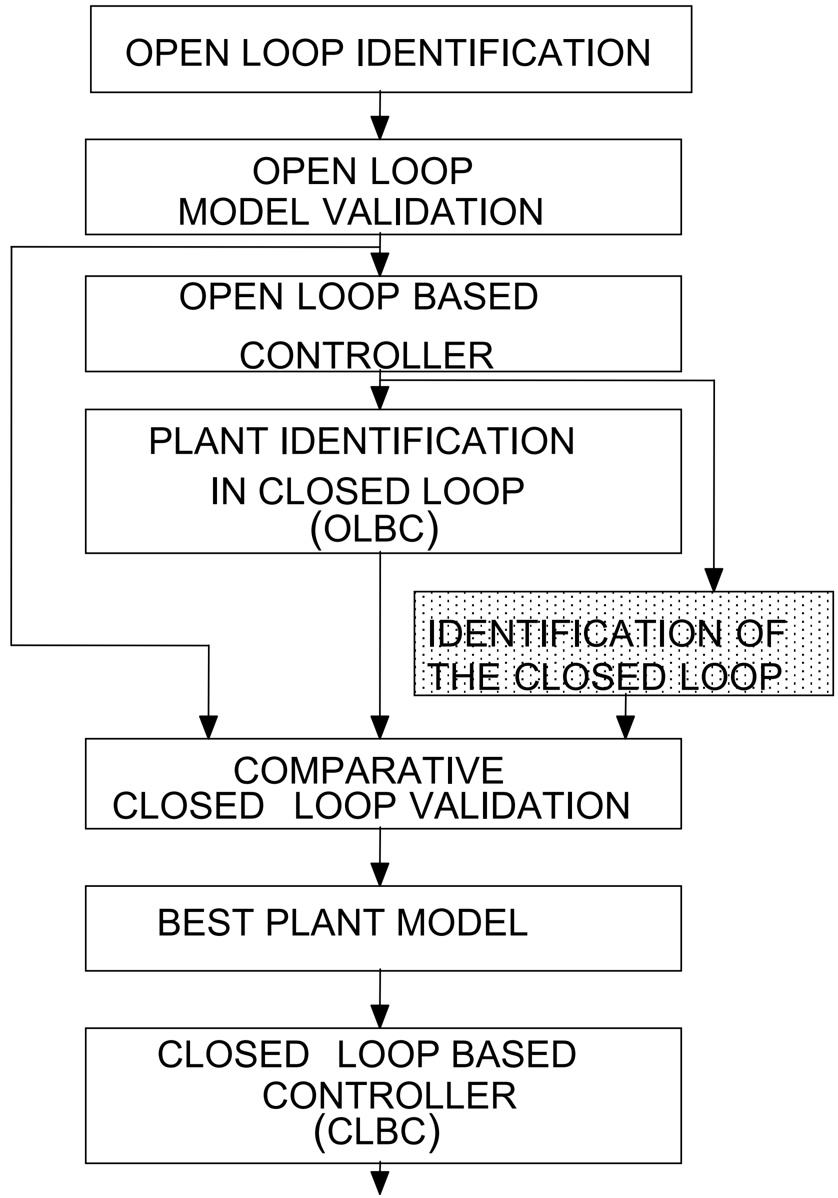
Comparison of « achieved » and « simulated » performance in the time domain



Rem.:

- not enough accuracy in many cases
- difficult interpretation of the results in some cases

Methodology of Plant Model Identification in Closed Loop



Closed loop identification schemes

Two possibilities for error generation:
-output error (CLOE)
-input error (CLIE)

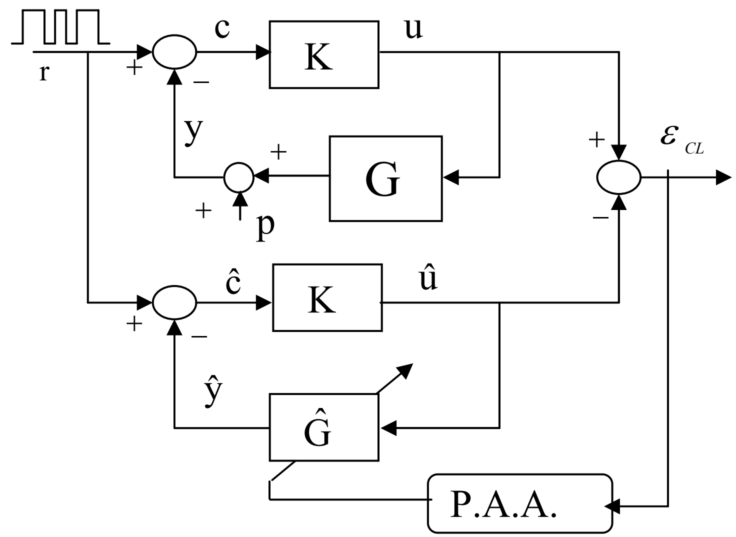
Two possibilities for applying the external excitation:
-added to the controller input (reference)
-added to the plant input

What is in fact important ?
The nominal sensitivity function we would like to approximate by the closed loop predictor (the identification criterion)

Remark:
Once a scheme is selected, to process the data one can use all the versions of the algorithms (choice of the regressor vector)

Closed loop input error (CLIE)

Excitation added to controller input (reference)



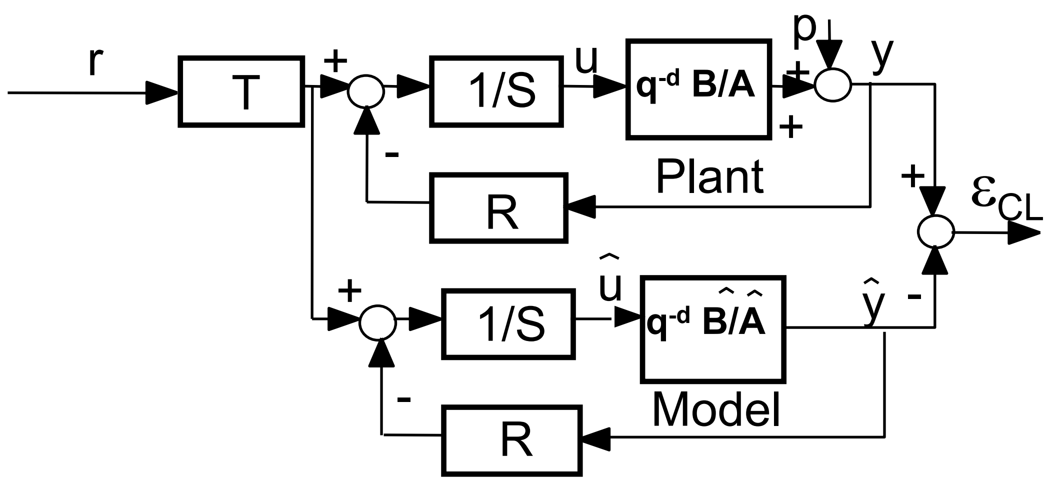
For details, see:

Landau I.D., Karimi A., (2002) : « A unified approach to closed-loop plant identification and direct controller reduction », *European Journal of Control*, vol.8, no.6

Selection of closed loop identification schemes

Identification criterion	Closed loop identification scheme
$\min\left\lVert S_{yp} - \hat{S}_{yp} \right\rVert$ or $\min\left\lVert S_{yr} - \hat{S}_{yr} \right\rVert$	CLOE with external excitation added to the controller input equivalent to CLIE with external excitation added to the plant input
$\min\left\lVert S_{up} - \hat{S}_{up} \right\rVert$	CLIE with external excitation added to the controller input
$\min\left\lVert S_{yv} - \hat{S}_{yv} \right\rVert$	CLOE with external excitation added to the controller output

Iterative Identification in Closed Loop and Controller Re-Design



Step 1 : Identification in Closed Loop

-Keep controller constant

-Identify a new model such that ε_{CL}

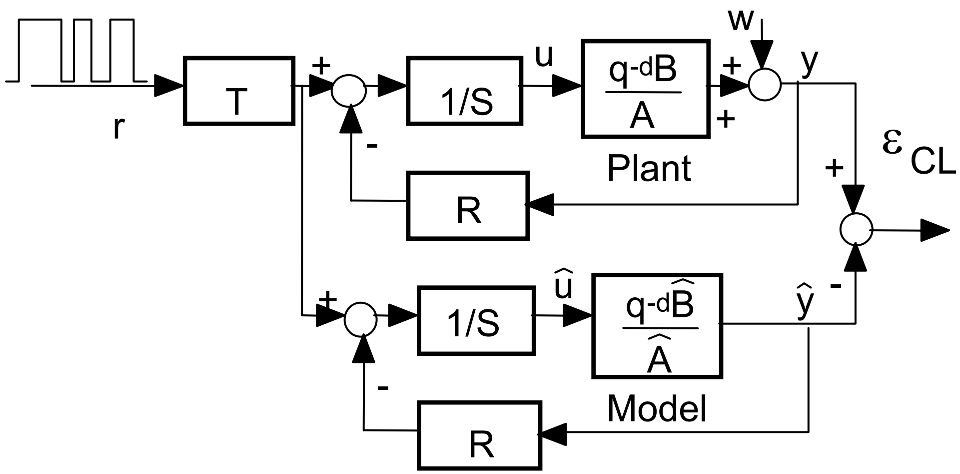
Step 2 : Controller Re – Design

- Compute a new controller such that ε_{CL}

Repeat 1, 2, 1, 2, 1, 2,…

An interesting connection CL/OL

Open loop identification algorithms are particular cases of closed loop identification algorithms



Use R=0, S=T=1 and you get the open loop identification algorithms

CLOE → (OL)OE

X-CLOE → X(OL)OE

Experimental Results

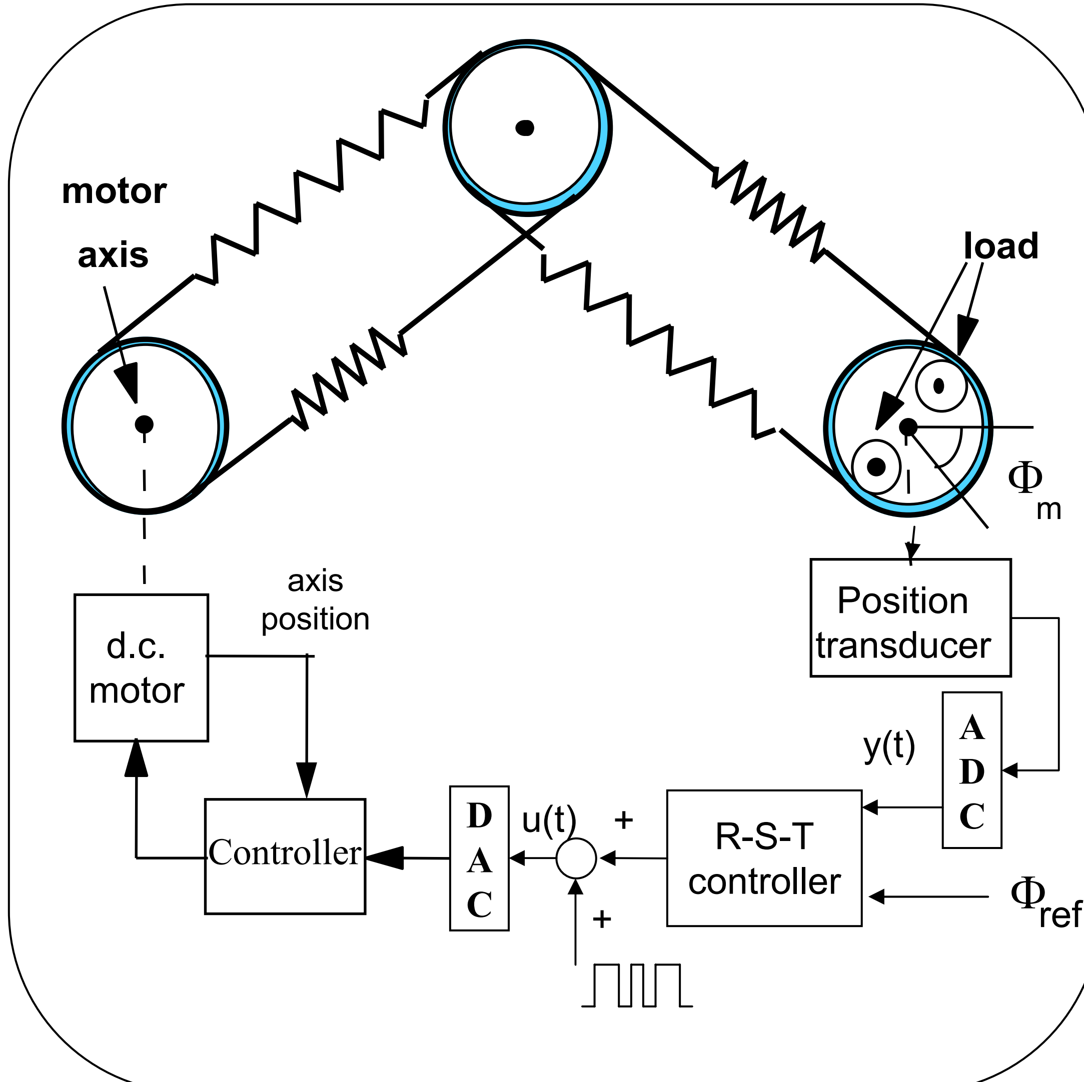
Identification in closed loop and controller re-design

for

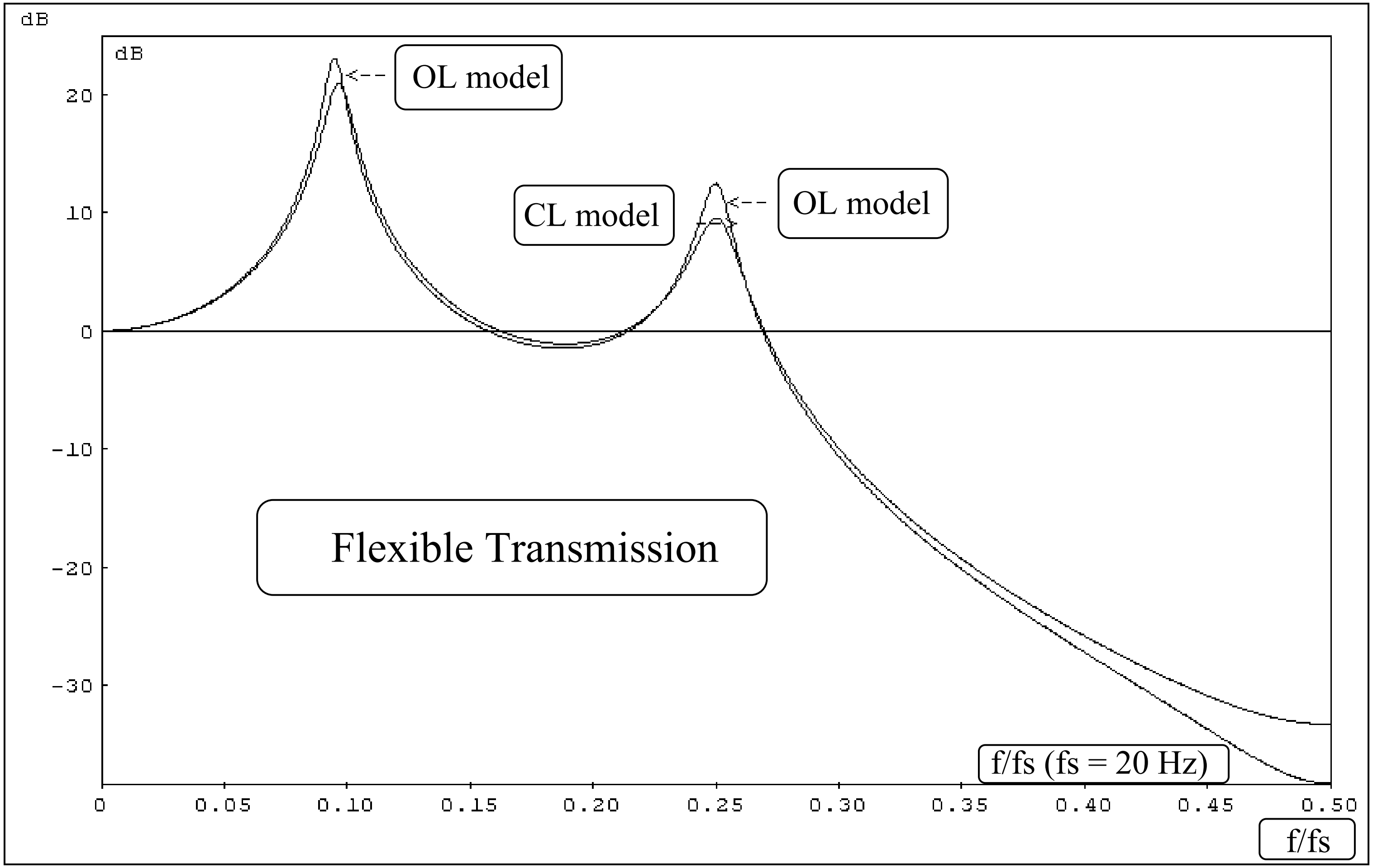
a flexible transmission

Identification in Closed Loop

The flexible transmission



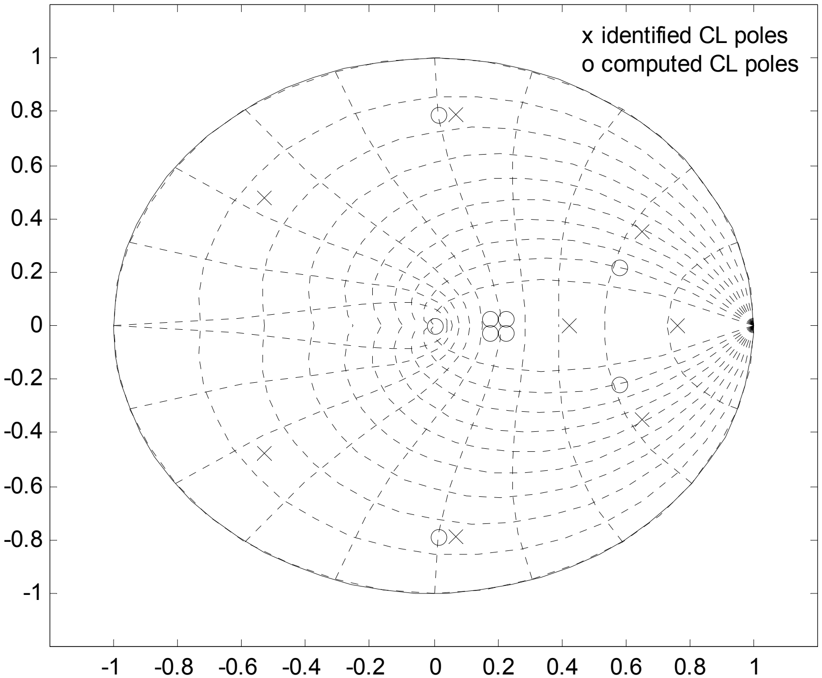
Flexible Tranmission
Frequency Characteristics of the Identified Models

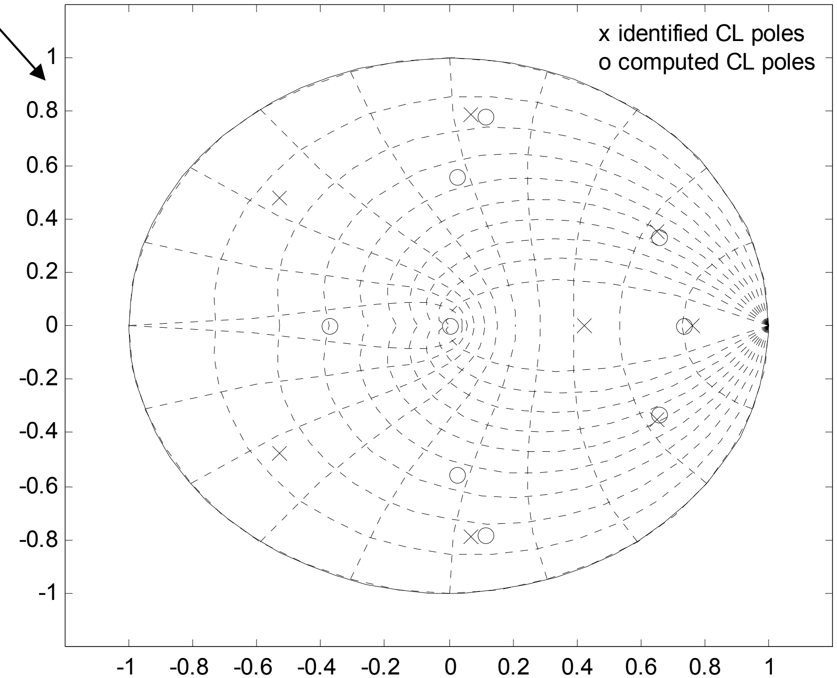


Model Validation in Closed Loop

Poles Closeness Validation

Controller computed using open loop identified controller (OLBC)





Model identified in open loop

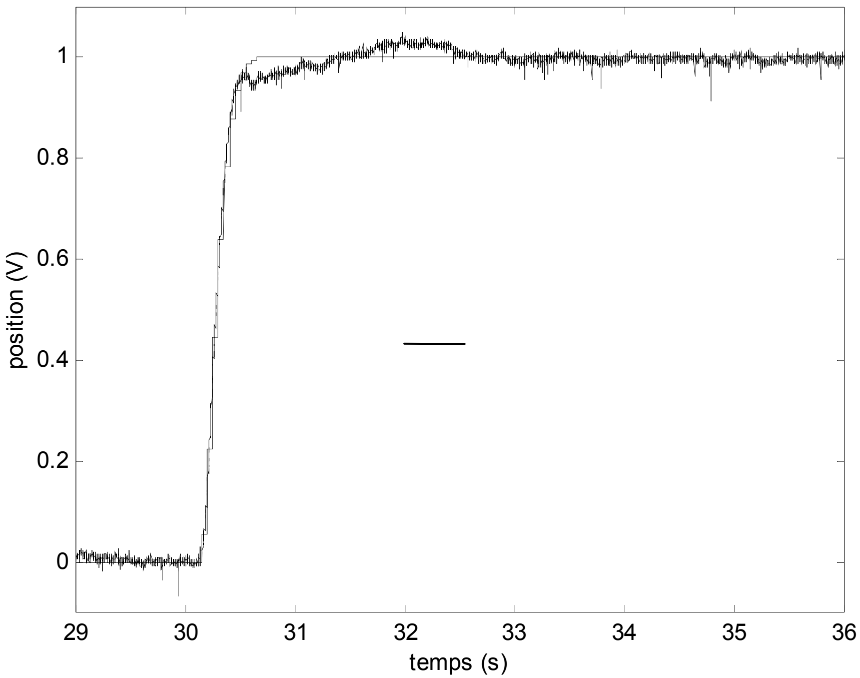
Model identified in closed loop

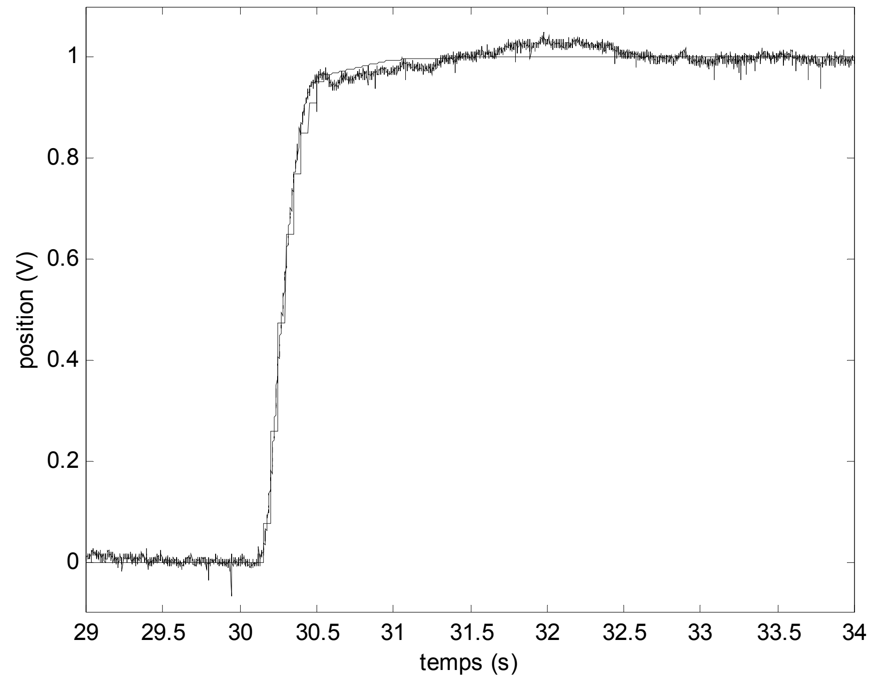
The model identified in closed loop provides « computed » poles closer to the « real » poles than the model identified open loop

Model Validation in Closed Loop

Time Domain Validation

O.L.B.C.



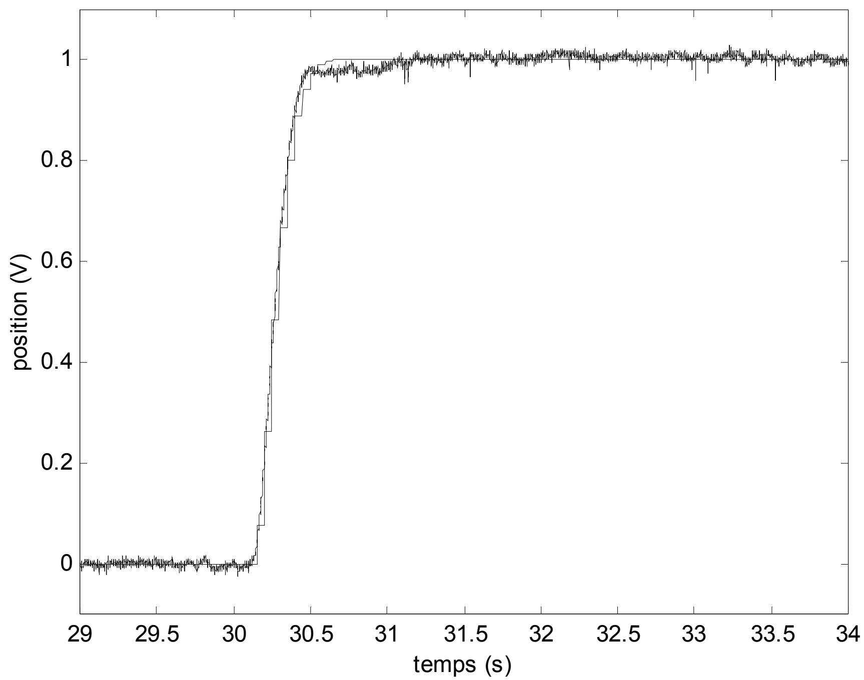


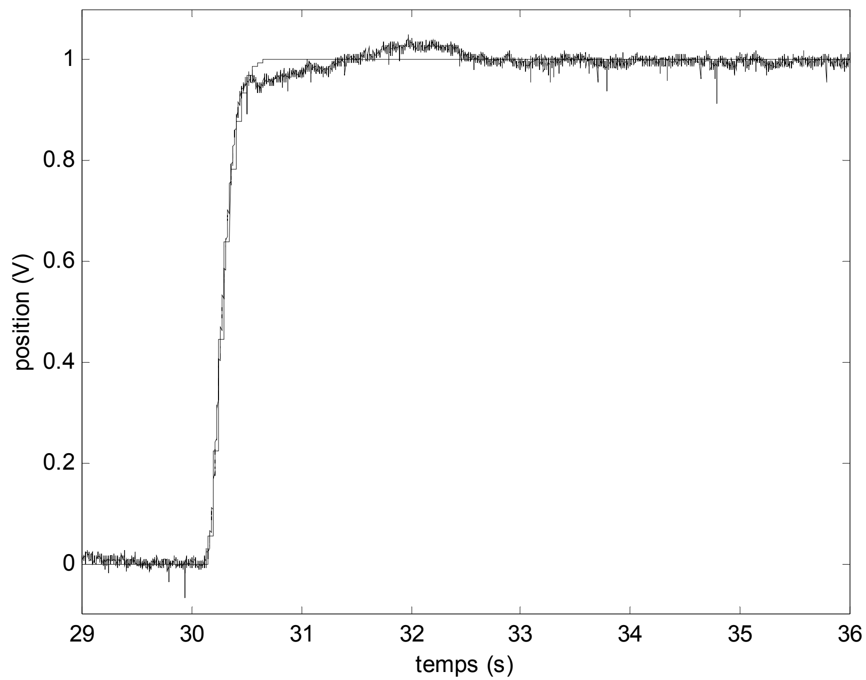
Model identified in open loop Model identified in closed loop

The simulation using the model identified in C.L. is closer to the real response than the simulation using the O.L.identified model

Controller Re-design Based on the Model Identified in Closed Loop

(on-site controller re-tuning)





Re-designed controller (CLBC) Initial controller (OLBC)

The CLBC controller provides performance which is closer to the designed performance than that provided by the OLBC controller

CLID™
(Matlab) Toolbox for Closed Loop Identification

To be downloaded from the web site:
http//:landau-bookic.lag.ensieg.inpg.fr

- files(.p and.m)
- examples (type :democlid)
- help.htm files (condensed manual)

CLID Toolbox

>> help clid

CLOSED LOOP IDENTIFICATION MODULE
by :
ADAPTECH
4 rue du Tour de l'Eau, 38400 Saint Martin d'Heres, France
info@adaptech.com

List of functions

cloe - Closed Loop Output Error Identification

fcloe - Filtered Closed Loop Output Error Identification

afcloe - Adaptive Filtered Closed Loop Output Error Identification

xcloe - Extended Closed Output Error Identification

clvalid - Validation of Models Identified in Closed Loop using also Vinnicombe gap

clie - Closed Loop Input Error Identification

>> help cloe

CLOE is used to identify a discrete time model of a plant operating in closed-loop with an RST controller based on the CLOE method.

[B,A]=cloe(y,r,na,nb,d,R,S,T,Fin,lam1,lam0)

y and r are the column vectors containing respectively the output and the excitation signal.

na, nb are the order of the polynomials A,B and d is the pure time delay

R, S and T are the column vectors containing the parameters of a two degree of freedom controller. S*u(t)=-R*y(t)+T*r(t)
Remark: when the excitation signal is added to the measured output (i.e. the controller is in feedforward with unit feedback) we have T=R and when the excitation signal is added to the control input (i.e. the controller is in feedback) we have T=S.

Fin is the initial gain F0=Fin*(na+nb)*eye(na+nb) (Fin=1000 by default)

lam1 and lam0 make different adaptation algoritms as follows:

lam1=1;lam0=1 :decreasing gain (default algorithm)
0.95<lam1<1;lam0=1 :decreasing gain with fixed forgetting factor
0.95<lam1,lam0<1 :decreasing gain with variable forgetting factor

See also FCLOE, AFCLOE, XCLOE and CLVALID.

CLOE – closed loop output error identification function

>> help cloe

Model Polynomials (identified)

Model orders

Initial adaption gain (Fin=1000 by default)

$$[B,A]=\text{cloe}(y,r,na,nb,d,R,\underbrace{S,T}_{\text{Controller Polynomials}},Fin,lam1,lam0)$$

Measured output

Excitation

Controller Polynomials

λ_1 λ_0

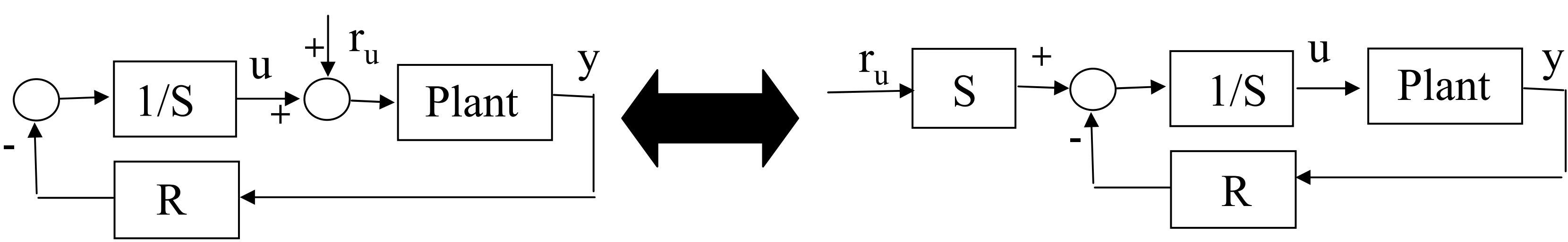
lam1=1;lam0=1 : decreasing gain (default algorithm)

0.95<lam1<1; lam0=1 : decreasing gain with fixed forgetting factor

0.95<lam1,lam0<1 : decreasing gain with variable forgetting factor

- Excitation superposed to the reference: ***Need to specify R, S and T***

- Excitation superposed to the controller output (i.e. plant input): ***Need to take T=S***



CLVALID – closed loop model validation function

\>> help clvalid

$$[\text{lossf},\text{gap},\text{Pcal},\text{Piden},\text{yhat}]=\text{clvalid}(\underbrace{B,A}_{\substack{\text{Model}\\ \text{Polynomials}\\ \text{(identified)}}},R,\underbrace{S,T}_{\substack{\text{Controller}\\ \text{Polynomials}}},y,r,\text{pcl})$$

lossf : $\frac{1}{N}\sum_{1}^{N}\left[y(t)-\hat{y}(t)\right]^2$

gap : Vinnicombe gap metric between *identified* and *computed* closed loop transfer function (BT/P)

Pcal : Computed closed loop poles by given model and controller

Piden : Identified closed loop poles from [y r] data

yhat : Closed loop estimated output

pcl = 1 : performs pole closeness validation (poles map display)
pcl =0; default

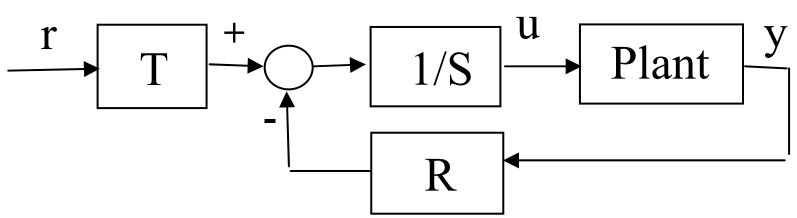
Display also the normalized crosscorelations $(r,\hat{y})$
Plots : cross correlations, autocorrelations (validation of the closed loop identified model), identified and computed closed loop transfer function, identified and computed poles

DEMOCLID –demo function

>> democlid

Load excitation(r) (PRBS), output data (y) controller and values for Fin (=1000), lam1(= 1), lam0(= 0)

Data are generated in closed loop with a RST controller
The external excitation is superposed to the reference



Plant model
for data generation
(simubf4.mat)
(r,y,u)

$$A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t) + C(q^{-1})e(t)$$

$$A(q^{-1}) = 1 - 1.5q^{-1} + 0.7q^{-2} \; ; \; B(q^{-1}) = q^{-1} + 0.5q^{-2} \; ; \; d = 0$$

$$C(q^{-1}) = 1 + 1.6q^{-1} + 0.9q^{-2}$$

Controller
for data generation
(simubf_rst.mat)

$$R(q^{-1}) = 0.8659 - 1.2763q^{-1} + 0.5204q^{-2}$$

$$S(q^{-1}) = 1 - 0.6283q^{-1} - 0.3717q^{-2}$$

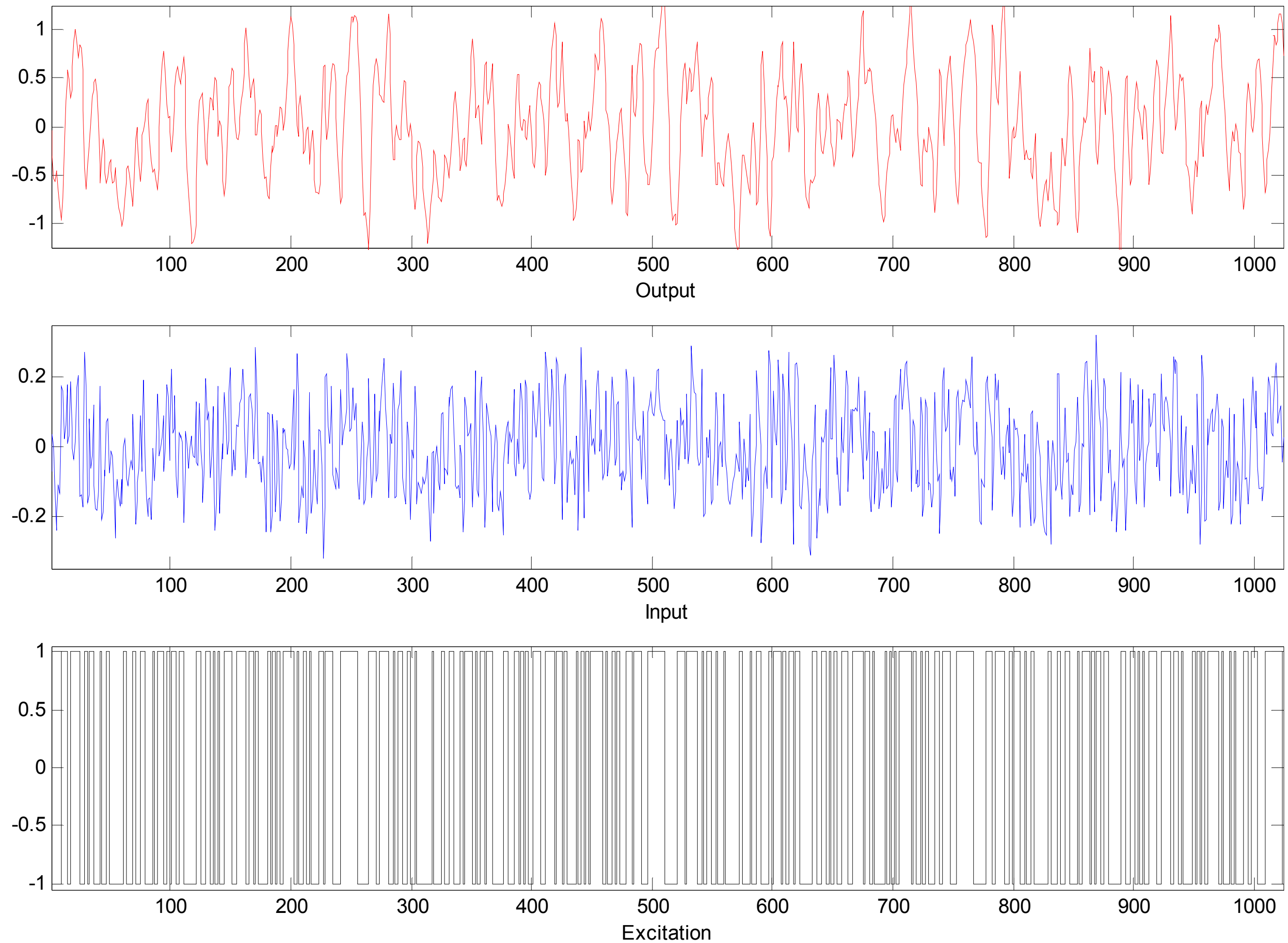
$$T(q^{-1}) = 0.11$$

File SIMUBF

Output (y)

Plant input (u)

External excitation (r)



Excitation superposed to the reference

Identification results

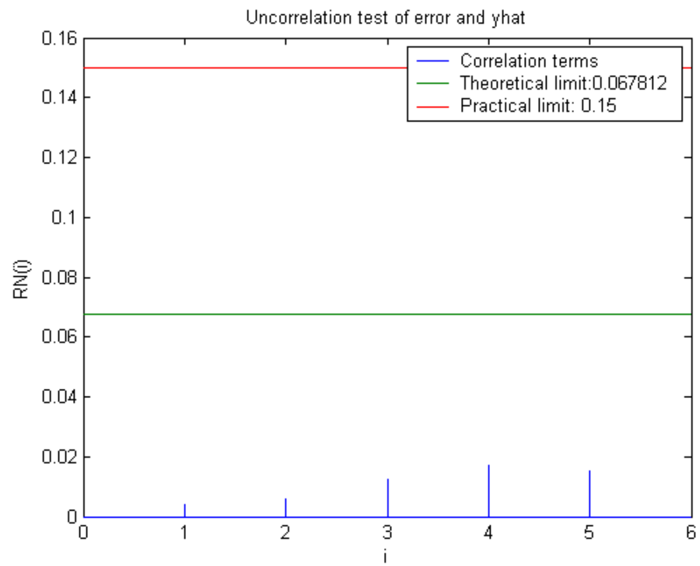
CLOE

```
>> [B,A]=cloe(y,r,na,nb,d,R,S,T,Fin,lam1,lam0)

B =

      0    0.9527    0.4900


A =

    1.0000   -1.4808    0.6716
```

AF-CLOE

```
>> [B,A]=afcloe(y,r,na,nb,d,R,S,T,Fin,lam1,lam0)

B =

      0    0.9684    0.4722


A =

    1.0000   -1.4844    0.6821
```

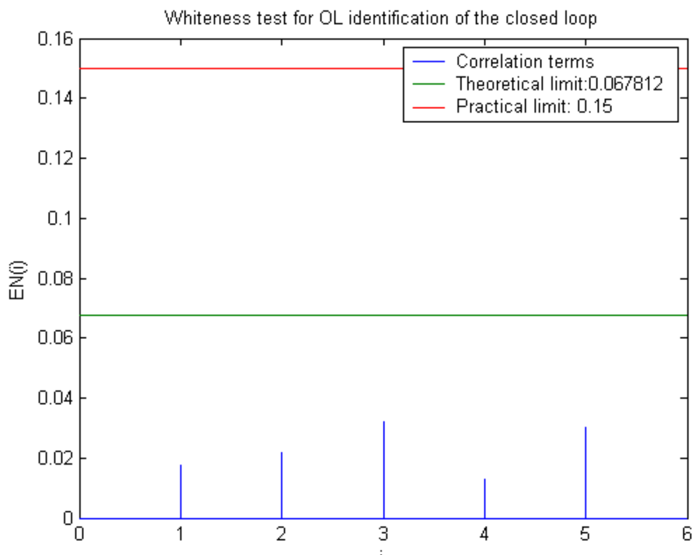
Statistical Validation of the model identified in closed loop



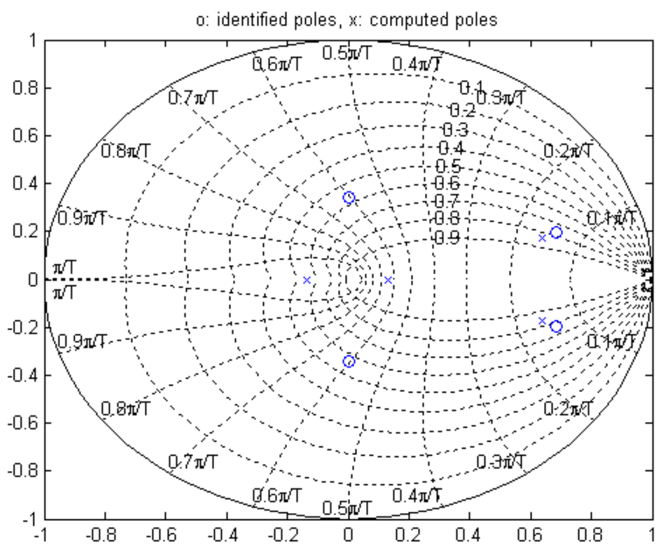
Model of the plant identified in closed loop with AF-CLOE

Is OK. $$\left|RN(i)\right| \leq \frac{2.17}{\sqrt{N}} = \frac{2.17}{\sqrt{1024}} = 0.0678; \quad i \geq 1$$

Poles closeness validation and ν-gap validation



Stochactic validation of the identified model of the closed loop
Is OK.
Can be used for poles closeness validation and ν-gap validation



Poles map
o identifed poles of the true CL system
x computed poles of the simulated CL system
Is OK

ν–gap = 0.0105 (min = 0, max =1)

File SIMUBF – comparison of identified models

Closed Loop Statisitical Validation

Méthod	a_1	a_2	b_1	b_2	CL Error Varaince R(0)	Normalized Intercorrelations (validation bound 0.068) |RN(max)|.
Nominal Model	-1.5	0.7	1	0.5		
AF-CLOE	-1.4689	0.6699	0.991	0.5276	0.03176	0.0092
CLOE	-1.476	0.6674	0.9592	0.4862	0.03181	0.0284
F-CLOE	-1.4692	0.6704	0.9591	0.5152	0.03175	0.0085
X-CLOE	-1.49	0.6822	0.9668	0.3775	0.0312	0.0237
OL type identification (RLS.)	-1.3991	0.6034	0.975	0.508	0.0323	0.0843

Best results

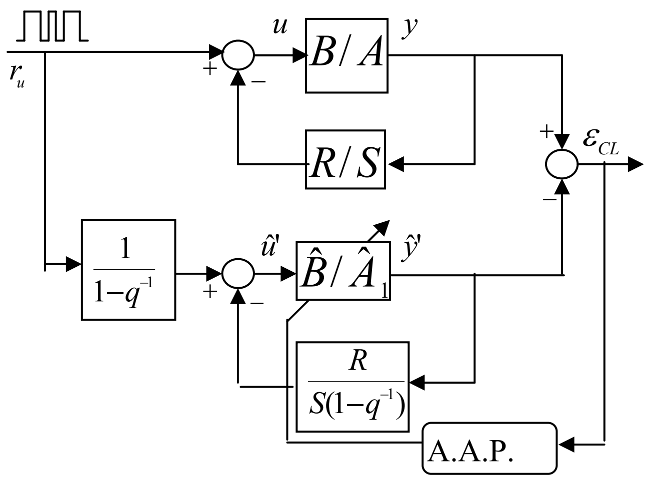
Open loop type identification
(between u and y ignoring the controller)

Concluding Remarks

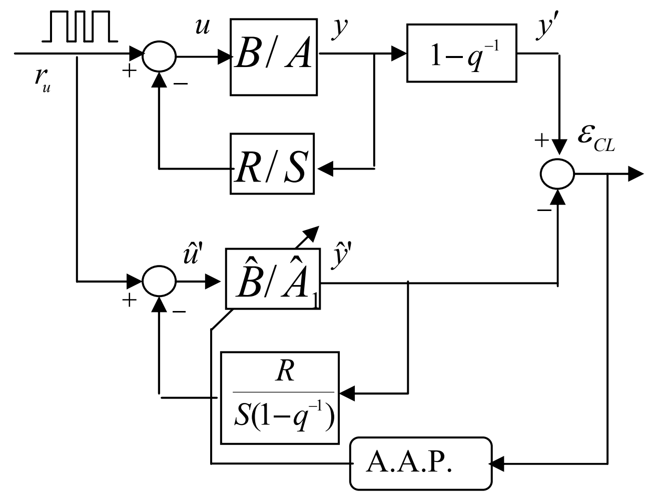
- Methods are available for efficient identification in closed loop
- CLOE algorithms provide unbiased parameter estimates
- CLOE provides “control oriented “reduced order” models (precision enhanced in the critical frequency regions for control)
-The knowledge of the controller is necessary (for CLOE and FOL)
-**In many cases the models identified in closed loop allow to improve the closed loop performance**
-**For controller re-tuning, opening the loop is no more necessary**
-Identification in closed loop can be used for “model reduction”
-By duality arguments one can use the algorithms for controller reduction
- **Successful use in practice**
- A MATLAB Toolbox is available (CLID- see website))
-A stand alone software is available (WinPIM/Adaptech)

Appendix

How to identify in closed loop systems with integrators ?



Replace the input of the closed loop predicor by its integral



Replace the measured output by its variations

Attention :

- the controller in the predictor has to be modified
- one identifies the plant model without integrator